N64-18132 gm
CODE-1
NASA CR-53467

97p.

OPTICAL SPACE COMMUNICATIONS

SYSTEM STUDY

FINAL REPORT

VOLUME II

SYSTEM TOPICS-PART ONE

OTS PRICE

XEROX $ 5.60 ph

MICROFILM $ 3.11 mf.

CONTRACT NO. NASw-540
NATIONAL AERONAUTICS AND
SPACE ADMINISTRATION

SPACECRAFT DEPARTMENT
GENERAL ELECTRIC

2#

OPTICAL SPACE COMMUNICATIONS SYSTEM STUDY.

FINAL REPORT

VOLUME II :

SYSTEM TOPICS - PART ONE

(NASA Contract ~~No~~. NAS w-540)

Office of Advanced Research and Technology

Headquarters

National Aeronautics and Space Administration

0 authors ? Feb~~ruary~~, 1964 97 p refs

(NASA CR-53467) OTS: $8.60 ph, $3.11 mf

Spacecraft Department
Missile and Space Division
0998914 General Electric Co~~mpany~~
Valley Forge Space Technology Center
P. O. Box 8555
Philadelphia, Pa., 19101

TABLE OF CONTENTS

INTRODUCTION

VOLUME II AND III

Volumes II and III, System Topics, presents the basis for the final system analysis, Volume IV, and the recommendations and conclusions, Volume I. Volume II reviews the results of a systems analysis of three space communication missions:

Mars-Earth Terminal
Mars-Earth Satellite
Moon Base-Earth Terminal

This analysis was based on the use of a quantum counter receiver and other available components, and serves as a reference for the advanced systems studied in Volume IV.

Volume II also deals with a number of derivations and calculations which are necessary for component evaluation. These include:

- A comparative analysis of PPM and PCM modulation with reference to peak/average power laser rating, available bandwidth, and means of generation.

- An analysis of collecting optics and passive optical filters, showing the relationships between SNR and collector size, collector quality, and filter resolution.

- An analysis of transmission through clouds.

- An analysis of the problem of wave front distortion in heterodyne detection, including an estimate of the severity of the problem and a proposed solution.

Volume II then presents a theoretical treatment of the information capacity of a noise quantized wave, and compares the capacities of heterodyne, homodyne, and quantum counter receivers to extract information from such a wave.

Volume III covers two main topics:

- Photomultiplier performance
- Atmospheric effects on laser propagation

It summarizes the results of tests conducted at the General Electric Advanced Technology Laboratories on star observations, photomultiplier performance, and laser propagation over a test range. The collecting optics quality and filter performance investigation of Volume II is given further treatment.

SECTION 1

MISSION ANALYSIS

Three space communications missions were analyzed:

- Mars to Earth Terminal
- Mars to Earth Satellite
- Moon Base to Earth Terminal

In the analyses, available components were postulated. Detection was by quantum counting, followed by an integrator with bit synchronization.

I. MARS TO EARTH TERMINAL

A. ASSUMPTIONS

Sky noise: Radiance at 7000 Å = 3×10^{-4} w/m^2 ster
Total Radiance = 4 w/m^2 ster
Mars noise: Radiance at 7000 Å = 3×10^{-3} w/m^2 ster
Total Radiance = 15 w/m^2 ster
(Receiver acceptance angle 3×10^{-5} radians
Aperture 5 m
(Receiver acceptance angle 10^{-4} radians
Aperture 10 m
Bit rate = 2.5×10^{7} bits/sec
Error rate = 10^{-4}
Filter Characteristics: Transmission @ 7000 Å = 0.1
Transmission @ 4000-8000 Å = 10^{-6}
Bandwidth = 1 Å
ENI = 10^{-12} watts (detector noise referred to input)
Quantum efficiency = 1%
Receiver optics efficiency = 50%

B. RECEIVER PARAMETERS

The detector characteristics require some detailed explanation. First, the detector is assumed to be an integrator which operates during a time period when a pulse may be present. At the end of the time period, the detector decides whether a pulse has been received. Errors occur when the detector fails to indicate a pulse or when noise is mistaken for a pulse. It will be assumed that the detector has prior knowledge of the pulse rate and synchronization. This assumption makes the error probability curves applicable to systems with an on time of less than 0.5. For a maximum bit rate of 2.5×10^7 bits/sec, a pulse width of .04 u sec. will be selected.

Assuming a high quality optical system with a five meter aperture limitation and a bit rate of 2.5×10^7 bits/sec, the detector input noise is $\bar{N}_N = 3.1 \times 10^{-3}$ photoelectrons per pulse. The total noise $\bar{N}_T = \bar{N}_N + N_S$ due to both internal and background noise is,

$$\bar{N}_T = 3.7 \times 10^{-3} \text{ photoelectrons per pulse}$$

This number is not physically real, but it may be interpreted as the probability of $\bar{N}_T = 1$. The Poisson distribution is

$$P_e = 0.5\left[\sum_{r}^{\infty} \frac{e^{-\bar{N}_T}\,\bar{N}_T^{\,Y}}{Y!} + \sum_{0}^{r} \frac{e^{-\bar{N}_S}\,\bar{N}_S^{\,Y}}{Y!}\right]$$

If the receiver threshold r = 2, then the first term becomes negligible and N_S may be chosen for the desired error rate. For $P_e = 10^{-4}$, $\bar{N}_S = 13$. Reversing the conversion procedure and assuming the aperture $D_r = 5$m, the received signal irradiance is

$$2 \times 10^{-9} \text{ watts/m}^2$$

Calculations will be continued for smaller bit rates; however, some assumption must be made concerning the pulse width. In the calculation of requirements for 25×10^6 bits/sec, the signal irradiance is large. This number expresses the fact that the required number of signal photons must arrive in a very short time. For progressively smaller bit rates, it would be possible to increase the pulse width and reduce the aperture. There is, however, a point at which average transmitter power becomes significant. For example, with low bit rates, a small pulse width results in a smaller average power compared to the equally on-off transmitter. Another consideration is detector internal noise. This noise will become large at low bit rates with long detector integration times. For purposes of analysis, the pulse width will be increased to the point where internal noise becomes significant.

C. NOISE COMPONENTS

The noise components are,

3.3×10^{-3} watts/m^2 ster A 7000 Å
19 watts/m^2 ster Total Radiance

The noise irradiance for $\theta_R = 3 \times 10^{-5}$ radians is

$$3.3 \times 10^{-3} (\pi/4) (3\times10^{-5})^2 = 2.3 \times 10^{-12} \text{ watts/m}^2 \text{ at 7000 Å}$$
$$19 (\pi/4) (3\times10^{-5})^2 = 1.3 \times 10^{-8} \text{ watts/m}^2 \text{ total noise radiance}$$

At the filter output,

$$P^1_{N1} = 0.1 (2.3 \times 10^{-12}) + 10^{-6} (1.3 \times 10^{-8})$$

$$P^1_{N1} = 2.4 \times 10^{-13} \text{ watts/m}^2$$

For $D_r = 5$ m

$$P_{N1} = 2.4 \times 10^{-13} (25)(.785) = 4.7 \times 10^{-12} \text{ watts}$$

In terms of photons

$$\frac{4.7 \times 10^{-12} \text{ joule/sec}}{3 \times 10^{-19} \text{ joule/photon}} = 1.6 \times 10^7 \text{ photons/sec}$$

Taking account of quantum and receiver efficiency,

$$1.6 \times 10^7 (.5)(.01) = 8 \times 10^4 \text{ photoelectrons/sec}$$

For an integration time $t = 4 \times 10^{-8}$ sec

$$\bar{N}_N = 8 \times 10^4 (4 \times 10^{-8})$$

$$N_N = 3.2 \times 10^{-3} \text{ photoelectrons}$$

The ENI is 10^{-12} watts. Converting,

$$\overline{N}_i = \frac{10^{-12} (5 \times 10^{-3}) (4 \times 10^{-8})}{3 \times 10^{-19}}$$

$$\overline{N}_i = 6.6 \times 10^{-4} \text{ photoelectrons}$$

The probability of error is

$$P_e = \tfrac{1}{2}\left[\sum_{r=X}^{\infty} \frac{e^{-\overline{N}_T}\,\overline{N}_T^{\,r}}{r!} + \sum_{r=0}^{X} \frac{e^{-(\overline{N}_S + \overline{N}_T)}(\overline{N}_S + \overline{N}_T)^r}{r!}\right]$$

where X = receiver threshold

$$\overline{N}_T = \overline{N}_i + \overline{N}_N$$

In the parametric systems study, x was set equal to $\overline{N}_S/2$. This selection was arbitrary and is not intended to represent the optimum case. Preliminary analysis indicates that for large numbers of received signal photons, x should be less than $\overline{N}_S/2$. A mathematical expression for an optimum threshold is difficult because of the characteristics of the Poisson distribution. By referring to a set of probability tables such as "Tables of Terms of Poisson Distribution", GE-DSD, D. Van Nostrand Co., New York, 1962, a value of x may be chosen. This value should require a minimum number of detector signal photoelectrons. For $P_e = 10^{-4}$, $\overline{N}_S = 13$. When $\overline{N}_S$ is translated to the receiver input,

$$P_S = \frac{13 \ (3 \times 10^{-19} \text{ joule/photon})}{(5 \times 10^{-3} \text{ photoelectrons/photon}) \ (4 \times 10^{-8} \text{ sec})}$$

$$P_S = 1.95 \times 10^{-8} \text{ watts}$$

D. RESULTS

The transmitter beam width is taken as 10^{-4} radians (20 seconds). This is about half of the angle subtended by Earth at Mars. Results are given for two cases:

1. 5 m receiving aperture with 3×10^{-5} radian (6 seconds) acceptance angle (limited by atmospheric refraction)

2. 10 m receiving aperture with 10^{-4} radian (20 seconds) acceptance angle (limited by quality of large collector)

CASE A

Bit Rate (Bits/sec)	Transmitter Power (Watts) Peak	Average	Energy (Joules/pulse)
2.5×10^7	1.08×10^5	5×10^5	.04
10^6	5.3×10^4	2.7×10^4	.053
10^5	6.8×10^3	3.4×10^3	.068
10^4	"	3.4×10^2	"
10^3	"	3.4×10^1	"
10^2	"	3.4	"
10	"	0.34	"

CASE B

Bit Rate (Bits/sec)	Transmitter Power (Watts) Peak	Average	Energy (Joules/pulse)
2.5×10^7	3×10^5	1.5×10^5	.012
10^6	1.6×10^4	8×10^3	.016
10^5	3×10^3	1.5×10^3	.03
10^4	"	1.5×10^2	"
10^3	"	1.5×10^1	"
10^2	"	1.5	"
10	"	0.15	"

II. MARS TO EARTH SATELLITE

A. ASSUMPTIONS

Mars noise: Radiance at 7000 Å = 3×10^{-3} w/m^2 ster
Total Radiance = 15 w/m^2 ster
Receiver acceptance angle 10^{-6} radians (0.2 sec)
Receiver aperture 1 m
Bit rate 10 to 2.5×10^7 bits/sec
Error rate 10^{-4}
Filter Characteristics: Transmission @ 7000 Å = 0.1
Transmission @ 4000-8000 Å = 10^{-6}
Bandwidth = 1 Å
Detector equivalent noise input (ENI) 10^{-12} watts
Quantum efficiency = 1%

B. NOISE COMPONENTS

Background noise irradiance at the receiver will be due to the illuminated disk of Mars. The receiver look angle will be limited by either the pointing accuracy of the satellite or the diffraction limit of the optics. For a satellite such as OAO, the pointing accuracy is 5×10^{-7} radians.

$$\theta_R = 1.22 \frac{\lambda}{D} = 1.22 \left(\frac{7 \times 10^{-7}}{1}\right) = 8.5 \times 10^{-7} \text{ radians}$$

For $\theta_R = 10^{-6}$, the receiver would look at only a portion of Mars. The price which must be paid for this noise reduction is the inclusion of a scanning capability on the satellite.

The noise irradiance at the receiver is:

(1) 2.13×10^{-12} watts/m^2 at 7000 Å

$\theta_R = 3 \times 10^{-5}$ radians (total disk of Mars)

1.07×10^{-8} watts/m^2 total, $\theta_R = 3 \times 10^{-5}$

(2) 2.26×10^{-15} watts/m^2 at 7000 Å

$\theta_R = 10^{-6}$ (the diffraction limit)

1.18×10^{-11} watts/m^2 total, $\theta_R = 10^{-6}$

If the filter characteristics of section I are used, the noise irradiance is:

(1) 2.24×10^{-13} watts/m^2 $\theta_R = 3 \times 10^{-5}$ radians

(2) 2.4×10^{-16} watts/m^2 $\theta_R = 10^{-6}$ radians

For a look angle which includes the entire disk of Mars, the noise is the same as that present at an Earth terminal. If a look angle of 10^{-6} radians is chosen, the optics on the satellite will be within practical size limits.

Calculations will be made for a one meter aperture and a look angle of $\theta_R = 10^{-6}$ radians. This number has been selected to show the advantage of a diffraction limited receiver not restricted by the atmospheric resolution limits.

With relatively low background noise, the internal detector noise determines the required signal irradiance. A reduction in internal noise does not produce a significant reduction in signal irradiance. For example, a change in equivalent noise input from 10^{-12} to 10^{-15} watts changes the required signal by a factor of 0.85.

C. RESULTS

Transmitter beamwidth = 10^{-4} radians

Bit Rate (Bits/sec)	Transmitter Power (Watts) Peak	Transmitter Power (Watts) Average	Energy (joules/pulse)
2.5×10^7	1.3×10^7	6.5×10^6	0.52
10^6	5.7×10^5	2.9×10^5	0.57
10^5	6.7×10^4	3.3×10^4	0.67
10^4	"	3.3×10^3	"
10^3	"	3.3×10^2	"
10^2	"	3.3×10	"
10	"	3.3	"

III. MOON BASE TO EARTH TERMINAL

A. ASSUMPTIONS

Same as Mars-Earth Terminal

B. NOISE EVALUATION

Noise at the receiver will be due to reflected sunlight and day sky. If a resolution limit $\theta = 3 \times 10^{-5}$ radians is selected for the receiver look angle, the receiver parameters become the same as the Mars-Earth terminal. This is true because the brightness of Mars and the Moon are quite similar.

The results for the Mars-Earth analysis are tabulated below: For $\theta = 3 \times 10^{-5}$ radians, the total noise irradiance is,

$$P^1_{N1} = 2.4 \times 10^{-13} \text{ watts/m}^2$$

For an aperture $D_r = 5$ m, error rate $P_e = 10^{-4}$, a bit rate of 2.5×10^7, and a pulse width of 4×10^{-8} sec, the received signal irradiance $P^1_S = 2 \times 10^{-9} \frac{\text{watts}}{\text{m}^2}$.

For a look angle of 10^{-4} radians which applies to large aperture collectors, the noise must be calculated. The noise radiant intensities are:

Sky noise:	Radiance at 7000 Å = 3×10^{-4} watts/m^2 ster
	Total Radiance = 4 watts/m^2 ster
Moon:	Radiance at 7000 Å = 3.2×10^{-3} watts/m^2 ster
	Total Radiance = 17 watts/m^2 ster

For a look angle $\theta_R = 10^{-4}$, the noise components are

Irradiance at 7000 Å = 2.74×10^{-11} watts/m^2
Total Irradiance = 1.65×10^{-7} watts/m^2

At the filter output,

$$P^1_{N1} = .1\,(2.74 \times 10^{-11}) + 10^{-6}\,(1.65 \times 10^{-7})$$

$$P^1_{N1} = 2.9 \times 10^{-12} \text{ watts/m}^2$$

If $D_r = 10$ m

$$P_{N1} = 2.3 \times 10^{-10} \text{ watts}$$

Receiver Parameters

The detector is assumed to have the characteristics described in Section I.

At the detector output, for a bit rate of 2.5×10^7 bits/sec and $D_r = 10$ m

$$\overline{N}_N = 0.14 \text{ photoelectrons}$$

The internal noise is,

$$\bar{N}_i = 6 \times 10^{-4} \text{ photoelectrons}$$

For $P_e = 10^{-4}$

$$\bar{N}_S = 17 \text{ photoelectrons}$$

The received signal is

$$P_S = 2.6 \times 10^{-8} \text{ watts}$$

For $D_r = 10$ m

$$P_S^1 = 3.31 \times 10^{-10} \text{ watts/m}^2$$

Transmitter Requirements

The received signal irradiance must be translated to the transmitter. The following assumptions will be made:

Range in meters $R = 4.03 \times 10^8$m
Atmospheric transmission = 0.5
Filter transmission at the source frequency f = 0.1

$$\frac{P_S^1}{P_T^1} = \frac{f}{R^2}$$

where P_T^1 = Radiant intensity of the source (Watts/ster).

For $D_r = 5$ m $\theta_R = 3 \times 10^{-5}$

$$P_S^1 = 2 \times 10^{-9} \text{ watts/m}^2$$

then,

$$P_T^1 = 6.5 \times 10^9 \text{ watts/ster}$$

For $D_r = 10$ m, $\theta_R = 10^{-4}$, and

$$P_S^1 = 3.31 \times 10^{-10} \text{ watts/m}^2$$

$$P_T^1 = 1.1 \times 10^9 \text{ watts/ster}$$

A simple transmitter pointing mechanism would aim at the disk of the Earth. Full coverage of the Earth would be obtained with a transmitter beamwidth $\theta_t = 3 \times 10^{-2}$ radians. This coverage would allow communication with one Earth station for roughly twelve hours. A reduction in transmitter power would result if the transmitter could be pointed at a specific point on Earth. For example, a transmitter beamwidth of 10^{-3} radians would allow about two hours communication time with one Earth station. For $\theta_t = 3 \times 10^{-2}$ radians, 2.5×10^7 bits/sec, and pulse width 4×10^{-8} sec.:

9. RESULTS

D_r (m)	Transmitter Power (Watts) Peak	Average	Energy (Joules/pulse)
5	4.6×10^6	2.3×10^6	.19
10	7.8×10^5	3.9×10^5	.03

For $\theta_t = 10^{-3}$ radians, 2.5×10^7 bits/sec, and pulse width 4×10^{-8} sec,

5	5.1×10^3	2.6×10^3	2×10^{-4}
10	860	430	3.4×10^{-5}

SECTION 2

MODULATION--ADVANTAGE OF PULSE MODULATION, PCM VERSUS PPM

One factor influencing the choice of modulation type is the capacity of the laser in peak and average power. If average power above a few milliwatts is required by the mission, the choice of laser is restricted to the optically pumped solid host variety. These lasers are characterized by a high ratio of peak to average power output. This characteristic stems from a limitation in the available pumping lamps and from the threshold power of the laser materials. The threshold pumping power density is so high that it can barely be reached by imaging the most intense available continuous sources such as the high pressure mercury arc. Such sources cannot drive the laser at a level high above threshold as required for efficient operation. Intermittent sources, such as the Xenon arc lamp can do so if the duty cycle is low - say 1%. These lamps have a repetition rate limited to perhaps 100 pps because of electrode heating.

We have the problem of transmitting the desired information rate with a low duty cycle low pulse rate carrier. If the information rate is higher than the available pulse repetition rate, two methods are suggested.

The main pulse can be modulated by means of an external shutter or by controlled Q switching into a series of sub-pulses. This method has been used by GE to produce 1000 sub-pulses in a one millisecond main pulse. An external Pockel cell shutter was used. Intra pulse modulation can also be done by controlled Q switching; this is more efficient than an external modulator and relieves the problem of relaxation noise.

Another way of getting multiple information bits per main pulse is to use pulse position modulation. The information is encoded in the time spacing between pulses and is decoded by the receiver on the basis of a shared running time base. If there are N resolvable time spacings available, N levels or $\log_2 (N)$ bits can be transmitted per pulse.

We can compare two systems - PCM and PPM - on the following basis:

ORIGINAL PAGE IS
OF POOR QUALITY

1. PCM intra pulse with 50% duty cycle within the pulse.
2. PPM with 10^4 resolved time spaces per pulse.
3. PCM error rate of 10^{-4} (sum of error on noise and error on signal plus noise).
4. PPM error rate of 10^{-4} (sum of error on noise and total error on signal plus noise).
5. PPM pulse width equal to PCM sub-pulse width. (This implies equal Q-switch performance).
6. The average number of noise photoelectrons received during the pulse width is 5.

On this basis, the following table can be constructed, using Poisson statistics:

	PCM	PPM
Noise photoelectrons/pulse width	5	5
Optimum receiver threshold (photoelectrons)	17	22
Signal photoelectrons per pulse	38	41
Error rate on signal	10^{-4}	10^{-4}
Error rate on noise	10^{-4}	10^{-4} per pulse width 10^{-8} per pulse

The procedure was to pick a receiver threshold to yield the desired error rate on noise. For PCM the desired error rate on noise is 1 in 10^4. This requires a threshold of seventeen photoelectrons if an average of five noise photoelectrons are present in a pulse width. For PPM the desired error rate is 10^{-4} per transmitted pulse or 10^{-8} per pulse width since there are 10^4 pulse widths associated with each transmitted pulse. The threshold required for an error rate of 10^{-8} per pulse width is twenty-two photoelectrons. Given these threshold values, the signal photoelectrons per pulse are found to be thirty-eight and forty-one for PCM and PPM respectively. The transmitted pulse energies for the two cases stand in the same ratio. The transmitted information is two bits per pulse for PCM and $\log_2 10^4 = 13$ bits per pulse for PPM. Under our assumptions, then, the channel capacity is 13/41 = 0.32 bits per photoelectron for PPM and 2/38 = .053 bits/photoelectron for PCM, an advantage for PPM of six to one. The advantage for PPM will increase if N is increased. The limit on N is imposed by the available clock stability and by the on-board data storage capacity. A value of 10^6 corresponds to twenty bits per pulse and would be practical to implement. If this value is selected and if 100 pulses per second are available, a maximum data rate of 100 x 20 = 2000 bits per second is possible with PPM. This performance implies a pulse width of 10^{-8} seconds. Given the same pulse rate and sub pulse width, an intra-pulse PCM system would transmit 2 x 10^6 bits per second at a duty cycle of 1%.

Conclusions

Pulse modulation offers the following advantages with present device technology:

Permits low duty cycle operation. This is the only mode available with high average power lasers.

Provides discrimination against noise. This is especially important because of the lack of narrow band pre-detection filters. Narrow pulse reception is well suited to the event-counting mode of photomultiplier operation, where the noise is impulsive rather than Gaussian. These considerations apply with great force in the use of very large aperture optics and in cloud cover reception because background noise predominates and dispersive filters are ineffective.

Pulse modulation has the following disadvantages:

Intermittent transmission requires on board data storage.

The maximum data rate is limited in comparison to continuous tranmission. For a given modulator bandwidth, the penalty is for PCM in proportion to the off/on time.

PPM offers a power advantage over PCM. For this reason it may be preferable in some cases. However, the available data rate is much less than for PCM.

SECTION 3

COLLECTING OPTICS AND FILTERS

In an earth-space link, primary interest is in the earth based receiver and the vehicle based transmitter. This assumes that vehicle limitations on size and weight are more pressing than ground station economic or technological limitations.

The paramount receiver design goal is minimization of the required vehicle transmitter radiant intensity ($W\ st^{-1}$). This goal has two obvious means:

Increase information per received signal energy.

Increase collector area.

It will be shown that there is an interaction between land 2, which leads to an optimum collector area for minimum signal radiant intensity. This optimum can not yet be prescribed, but the factors involved have been identified.

The relationships are developed so that given a signal and noise radiant intensity, the SNR can be predicted in terms of the collecting optics aperture and field of view, and the size and dispersion of the filter.

The filter-optics interaction is explained in terms of optics geometry. The optic system has an entrance aperture D_o and a field of view of B_o radians, the latter determined either by a limiting stop, by collector quality, or by diffraction limit.

The optic system has an exit aperture D_1 and there is a vergence B_1 radians in the exit beam,

$$B_o\ D_o = B_1\ D_1$$

If the exit beam is reflected from a grating of dispersion α radians/Å, and focussed on a detector, the resolution line width of the system will be

$$\Delta\lambda = \frac{B_o}{\alpha}\ \frac{D_o}{D_1}\ \text{Ångstroms}$$

In practice, the grating aperture must be as large as D_1. This limits D_1 to the size of available gratings. It is evident that the resolution linewidth of the system is proportional to $B_o D_o$. In diffraction limited collectors, $B_o D_o$ is constant. In poorer quality collectors, B_o will increase with D_o so that the resolution linewidth will be in general worse for very large collectors.

If the transmitter is seen against an extended noise background which has constant spectral intensity in the filter linewidth, the SNR can be deduced.

The received signal power is,

$$P_s^1 \ (\frac{\pi D_o^2}{4})$$

The received post filtering noise power is,

$$P_{e2} \ (\frac{\pi D_o^2}{4}) \ (\frac{\pi B_o^2}{4}) \ (\frac{B_o D_o}{D_o})$$

The post filter SNR is

$$SNR = \frac{P_S^1}{P_{e2}} \ (\frac{4 \ D_1 \alpha}{\pi D_o B_o^3}), \text{ where}$$

P_S^1 = signal irradiance at receiver W M^{-2}
P_{e2} = noise spectral radiant intensity W M^{-2} st^{-1} $\mathring{A}^{-1}$

Consider three cases - B_o defined by atmospheric seeing, B_o defined by diffraction limit, and B_o defined by optics quality.

For atmospheric seeing, let B_o = 2 sec = 10^{-5} rad.

$$SNR = \frac{P_S^1}{P_{e2}} \ (\frac{4 \ D_1 \alpha}{\pi D_o \times 10^{-15}})$$

For diffraction limit $B_o = 1.2 \ \lambda / D_o$

$$SNR = \frac{P_S^1}{P_{e2}} \ (\frac{4 \alpha D_1 \ D_o^2}{\pi (1.2 \lambda)^3})$$

For optics quality the relationship would be quantitative if we had data relating B_o and D_o. For example if $B_o = K D_o^n$

$$SNR = \frac{P_S^1}{P_{e2}} \left(\frac{4 \; D_1 \alpha}{\pi K^3 \; D_o^{3 \; n+1}} \right)$$

It should be emphasized that the foregoing work applies only to systems using filters of the dispersive class, where the filter resolution linewidth is proportional to the entrance beam vergence. It can be applied to multiple layer interference filters if an appropriate expression for the vergence-resolution relation is substituted. In the case of coherent heterodyne detectors, a special relation exists between the output signal and the collimation of the incident signal. This and other special cases will be covered in later phases of the program.

The error rate is determined jointly by the SNR and the signal level. For a fixed error rate the required SNR decreases as the signal level increases. For example, from Section 2 we have the PCM signal level and SNR for an error rate of 10^{-4}. We will calculate the required signal for a noise level 22 times greater.

Noise photoelectrons/pulse width	5	110	1330
Receiver Threshold	17	150	1500
Signal photoelectrons/pulse width	38	90	320
Post filter SNR	7.6	0.82	.24
Error Rate	10^{-4}	10^{-4}	10^{-4}

This allowable increase in noise compared to signal implies that large low quality optics may be feasible.

SECTION 4

TRANSMISSION THROUGH CLOUDS

In receiving a laser transmission from space through a cloud layer, we are interested in the path loss as compared to the clear weather case. It is known that the sun illumination on an overcast day is from 1% to 10% of the sun illumination on a clear day, as measured by a wide angle photometer. Now suppose that a laser beam is striking the top of the cloud layer, and is wide enough to illuminate the cloud layer from horizon to horizon or beyond. Since this duplicates the situation obtaining in the sunlight case it is reasonable to assume that a wide angle collector would collect from 1 to 10% of the signal light that it would collect on a clear day. If the laser beam is much narrower, so that it illuminates a small part of the clouds visible from the receiver, the attenuation relative to clear sky conditions will be much greater. For example, if the laser beam were 100 times as wide as the receiver aperture, the clear sky attenuation would be 10^{-4}. But on a cloudy day the clouds would act at best as an isotropic lossless scatterer. In this case, if the clouds were at an altitude h, the fraction intercepted by a receiver of aperture area A would be $A/4\pi h^2$. For $h = 3000$ M, $A = 1\ M^2$, the attenuation is $1/4\pi \times 9 \times 10^6 = 8.8 \times 10^{-9}$, giving a minimum loss of 8.8×10^{-5} due to the clouds.

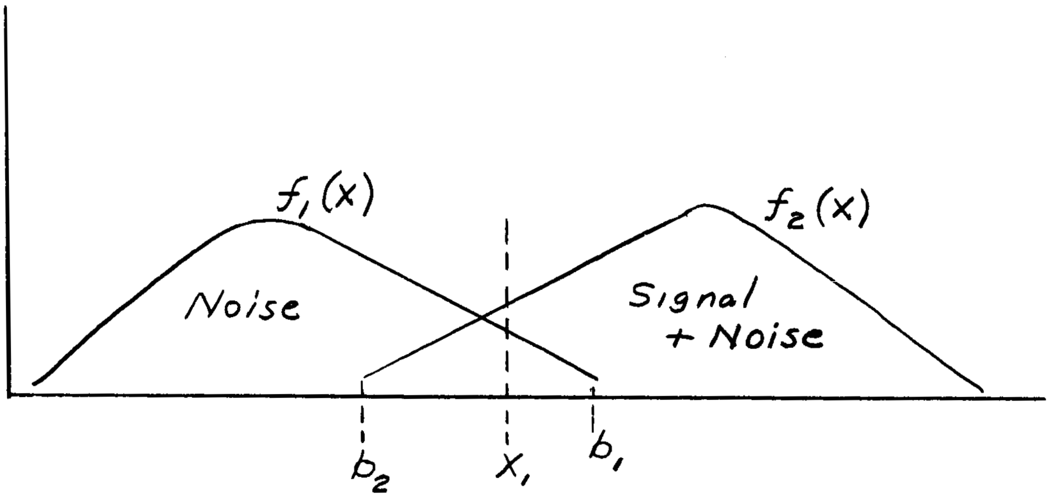
The presence of clouds imposes a severe requirement on the receiver optical filter. In daylight, the wide angle receiver will collect from 1 to 10% of the sunlight it would collect if aimed at the clear day sun. At night it will collect a similar proportion of the integrated night sky sources. Furthermore, the wide angle optical system severely penalizes the performance of any optical filter of the dispersive class, since these filters depend on a narrow field of view to obtain narrow band pass filtering. Another difficulty with wide angle optics is that the area of the photosensitive surface must be large enough to cover the image surface. For a system with a field of view of 1 radian, an aperture of 1 meter, and a numerical aperture of 0.5, the photosensitive surface must have an area of 1970 cm^2. Such a large photodetector would have relatively high dark current.

Finally, if the cloud layer is 3000 M high, the difference in path length from the center to the edge of a 1 radian field of view is 1500 feet, giving a delay spread of 1.5 microseconds. This will spread the received pulses and limit the pulse repetition rate and time resolution.

SECTION 5

OPTIMUM RECEIVER THRESHOLD

In the following analysis, it will be shown that the minimum error rate occurs when the receiver threshold is set at the intersection of the signal and noise distributions. This statement is based on the following proof.



The probability density functions $f_1(X)$ and $f_2(X)$ are restricted so that $f_1(X)$ is monotonically decreasing for $b_2 \leq X \leq b_1$ and $f_2(X)$ is monotonically increasing for $b_2 \leq X \leq b_1$. The total probability of error will be

$$P = \int_{X_1}^{b_1} f_1(X)\,dx + \int_{b_2}^{X_1} f_2(X)\,dx$$

where X_1 = receiver threshold

$$P = \Big[F_1(X)\Big]_{X_1}^{b_1} + \Big[F_2(X)\Big]_{b_2}^{X_1}$$

$$P = F_1(b_1) - F_1(X_1) + F_2(X_1) - F_2(b_2)$$

$$\frac{dP}{dX_1} = -F_1^{\,1}(X_1) + F_2^{\,1}(X_1)$$

$$\frac{dP}{dX_1} = -f_1'(X_1) + f_2'(X_1)$$

Let $\frac{dP}{dX_1} = 0$

Then, $f_1(X_1) = f_2(X_1)$

For Poisson distributed noise,

$$f_1(X) = \frac{e^{-a_1} a_1^X}{X!} \qquad f_2(X) = \frac{e^{-a_2} a_2^X}{X!}$$

Solving for X,

$$X = \frac{a_1 - a_2}{\ln \frac{a_1}{a_2}}$$

where $a_1 = \overline{N}_T$ = total number of noise photoelectrons

$a_2 = \overline{N}_T + \overline{N}_S$ = total number of signal and noise photoelectrons.

SECTION 6

NON SYNCHRONIZED PCM

In previous work it was assumed that the receiver was synchronized with the transmitter. The detector was assumed to operate during a time when a pulse was expected. This condition implied a knowledge of transmitter pulse width, bit rate, and relative phase. The case will now be considered where the receiver has prior knowledge of the pulse width only.

Assume a detector with an observation time equal to the pulse width. For a pulse width τ, and a bit rate 1/T, the error due to mistaking noise for a pulse is,

$$P'_{e1} = T/\tau \quad P^1_{e1}$$

where,

$$P^1_{e1} = \sum_{r=x}^{\infty} \frac{e^{-\bar{N}_T}\,\bar{N}_T{}^r}{r\,!}$$

$\underline{X}$ = receiver threshold
N_T = internal + background noise

This increase in the probability of error due to mistaking noise for signal occurs because the detector must make t/τ decisions regarding the presence of a pulse.

The passive integrator sums both signal and noise pulses until some threshold is reached. The contents of the integrator at any time is the weighted sum of past signal and noise events. The summing process may be represented by,

$$N = N_i \int_0^{n\tau} (1-e^{-t/\tau}) \, dt$$

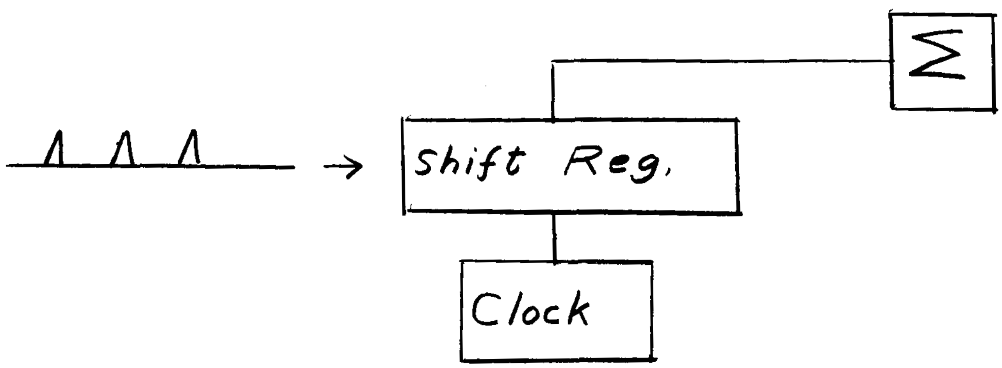
where N_i = number of noise photoelectrons per sec
n = number of observation time slots
t = time of a detector observation
τ = pulse width and time constant of the integrator

Since e^{-n} approaches zero rapidly, the total noise is approximately,

$$N = n \tau N_i$$

This result agrees with the noise encountered in an integrate and dump detector.

Another technique of detection involves the use of a shift register as a matched filter. The mechanism would have the following form:



The number of digits in the register is numerically equal to the detector threshold. The clock is adjusted to allow passage of a digit through the register in τ seconds. The input pulses result from signal and noise photoelectrons. The pulses may alternatively result from the output of a level discriminator which reduces the effect of detector internal noise. The probability of error due to noise will be the same as that in the passive integrator.

The signal required without bit synch will now be examined. An integrate and dump detector is not practical without bit synch because only a portion of the signal pulse may be integrated. The worst case occurs when only half the signal pulse is contained within any one observation time slot. With passive integration, the signal is,

$$\bar{N}_S = \bar{N}_{S1} \int_0^{\tau} (1 - e^{-t/\tau})\, dt$$

where $\bar{N}_{S1}$ = number of signal photoelectrons per second
t = time of a detector observation
τ = pulse width and time constant

If a signal pulse begins at $t = 0$, the integrated signal τ seconds later is,

$$\bar{N}_S + \bar{N}_{S1} \int_0^{\tau} (1 - e^{-t/\tau})\, dt$$

$$\bar{N}_S = 0.37 \quad \bar{N}_{S1}$$

Therefore, the signal must be increased by 60% to maintain an error rate equal to the bit synch case. For signal indication with a shift register, the signal requirements are the same as an integrate and dump detector with bit synch.

As an example of the effect of non-synchronous operation, a calculation from the Mars-Earth mission analysis will be repeated. For an error rate of 10^{-4}, a pulse width of 10^{-5} sec, and a bit rate of 10^5 bits/sec, it was found that $\bar{N}_S = 38$. Also $\bar{N}_T = 5$, $P^1_{e1} = 10^{-5}$, and a threshold $X = 17$. Then,

$$P_{e1} = .1/10^{-5}\ (10^{-5}) = .1$$

Since the noise is fixed, the receiver threshold must be increased to reduce P_{e1} to 10^{-4}. By referring to Poisson Tables, a threshold of $X = 22$ is found to be necessary. With this larger threshold, the signal must also be increased. From the tables,

$$\bar{N}_S + \bar{N}_T = 45$$

$$\bar{N}_S = 40 \text{ photoelectrons}$$

SECTION 7

CONCLUSIONS AND RECOMMENDATIONS
SECTIONS 1 THROUGH 6

1) An ideal optical system, operating in a realistic space environment, will outperform a radio system in a space-earth link by about 63 db on the basis of primary power consumption.

2) An optical system using available components will be about 33 db worse than a radio link on the same basis.

3) The principle means of improvement are:

Ground Collector Area
Pointing Accuracy
Laser Efficiency
Detector Quantum Efficiency
Filter Transmission

4) Pulsed modulation is most practical now because:

a. Higher average power lasers operate in pulsed mode.
b. Pulsed operation affords discrimination against background and internal noise which cannot be equalled with present filters.

5) Maximum advantage of very large collectors is contingent on development of new filters which are insensitive to entrance angle vergence. Heterodyne detection is not suitable from this aspect.

6) Pulsed modulation will restrict message bandwidth to the sub-megabit/second range. Efficient PCM or PPM modulators for such speeds must be developed; they will be of the controlled Q switch type.

7) Wider information bandwidths will require high power CW lasers. Much better filters will be needed than for pulse operation, or the heterodyne detector must live up to its promise.

8) Heterodyne detection has great potential. Two serious problems remain:

a. An acceptable conversion efficiency has not been demonstrated.
b. The requirement for parallel wavefronts in signal and L.O. is incompatible with any but the smallest collectors.

9) Transmission from space through clouds to earth imposes a rather small signal attenuation. It poses two problems:

a. The wide angle optics will collect a large amount of background noise. The noise will be presented to the filter with a large vergence. Dispersive or interference filters will be practically useless.
b. The wide angle optics require a large area photosensitive surface, comparable to the effective collecting area. This implies a problem with detector internal noise.

Up transmission through clouds is prohibited for most missions.

10) The use of the time coherent property of laser radiation is of limited value in photon counting detection systems. At best, a small improvement in SNR can be expected. For heterodyne detection, a strong discrimination is possible, both against off axis noise and against on-axis non-time coherent noise.

Recommendations

1) The use of large area collecting optics for ground receivers should be further investigated.

a. Determine resolution angle as a function of aperture for all feasible methods of fabrication up to apertures of 250 feet. Determine ranges of reflectance and scattering to be expected. Estimate costs where possible.

b. Use (a) to calculate optimum collector size with dispersive filters. Calculate filter specifications as a function of collector size.

c. Investigate the effect of collecting aperture size on amplitude modulation, dancing, and defocussing in the atmosphere. Relate this information to the preceding calculations.

2) Those filters which are independent of input angle vergence should be investigated. In particular, scattering filters, specular vapor filters, and gas absorption filters are potentially useful.

3) The systems aspects of heterodyne detection should be explored. In connection with the parallel wavefront requirement, the effect of atmospheric refraction and optics quality must be studied. There

is a possibility of noise discrimination by delayed autocorrelation (See tenth Conclusion). This should be evaluated.

4) Close monitoring of progress in heterodyne detection should continue. In view of the present lack of experimental data on low level detection, some experiments may be needed to clarify the situation.

5) Detector quantum efficiency should receive attention. In particular, means for improving the effective surface absorbtivity by external mirrors have been proposed and should be studied.

6) A survey of photoconductive detectors should be made in view of the importance this study has assigned to quantum efficiency compared to internal noise.

7) Because of the requirement for large photosensitive surfaces in large collecting optics systems, the design problems and performance of large area detectors should be studied.

8) In view of conclusion (6), means of efficient and rapid pulse code and pulse position modulation, such as controlled Q switching, should be studied.

9) The on-board pointing problem should be studied, to indicate the probable tradeoff between pointing equipment weight and incremental transmitter weight. This will establish at what power level a given refinement in pointing becomes feasible.

10) The selection of an optimum per-bit error rate is complicated by the unusual characteristic of noise-in-signal for optics. The use of redundant coding with per-bit error rate higher than message error rate should be studied.

SECTION 8

INFORMATION CAPACITY OF AN OPTICAL COMMUNICATION CHANNEL

The absolute information capacity of an optical communication channel is of interest becaust it sets an upper limit to the performance of an optical communication system and because it provides a measure of the efficiency of a given communication system.

There have been several approaches to the calculation of an optical channel capacity.[1,2,3] They are all based on the concept of information as entropy, introduced by Shannon.

This concept is illustrated by the famous equation

$$C = B \log_2 (1 + S/N), \text{ where} \qquad (1)$$

C is the capacity in bits per second of signal of average power S and bandwidth B, in the presence of additive noise power, N, providing that both S and N have the statistics of white noise.

In extending this relationship to optical waves, it is necessary to account for the presence of significant quantum energy, and to determine that the optical wave otherwise conforms to the conditions for validity of equation (1).

The information capacity of a wave is a function of its bandwidth, duration, total energy, and quantum energy.

If the wave is T seconds long and has a bandwidth B, it will contain BT modes or Nyquist intervals.

If the wave energy is E, the average energy per mode will be $\frac{E}{BT}$ and the average number of quanta per mode will be $\frac{E}{BTh\nu} = \bar{m}$

The total number of quanta in the wave is

$$\frac{E}{h\nu} = \bar{m}\, BT$$

The information capacity of the wave depends on how many different ways the quanta can be distributed among the BT modes, under the condition that the total number of quanta (wave energy) is fixed.

The entropy or information per mode, for a large number of modes is

$$H = -\sum_{m} p\ (m) \log p\ (m), \text{ where} \tag{2}$$

p (m) is the probability of just m quanta in a mode.

Equation (2) can be interpreted as an assertion that the information to be derived from an event (exactly m quanta in a mode) is equal to the probability of the event, p (m), times the logarithm of the number of ways the event could have occured, $-\log p\ (m) = \log 1/p\ (m)$.

As an illustration of this concept, the information content of a rather simple wave was calculated. It was assumed that the wave contained just four quanta distributed among four modes. There are 35 different ways of distributing the quanta, corresponding to an information capacity.

$$C_w = \log_2 35 \sim 5 \text{ bits}$$

Equation (2) is maximized when the distribution of quanta is

$$p\ (m) = \frac{1}{1 + \bar{m}} \left(\frac{\bar{m}}{1 + \bar{m}}\right)^m \tag{3}$$

This distribution maximizes H because it is the most random distribution consistent with an average power of $\bar{m}$ $Bh\nu$. It is also consistent with the exponental distribution of power in a wave of white noise.

Substituting (3) in (2)

$$H = \log (1+\bar{m}) + \bar{m} \log (1+/\bar{m}) \text{ bits per mode} \tag{4}$$

Since there are B modes per second,

$$R = HB = B \left(\log (1+\bar{m}) + \bar{m} \log (1+1/\bar{m})\right) \tag{5}$$

And substituting the relation $\bar{m} = P/h\nu B$

$$R = B \log (1+P/h\nu B) + P/h\nu \log (1+h\nu B/P) \quad \text{bits per second} \tag{6}$$

The foregoing analysis is taken from Reference 1. Reference 2 follows Reference 1 in deriving equation (2). However, Reference 2 considers that a wave of bandwidth B has 2 BT degrees of freedom, corresponding to an amplitude and phase measurement at intervals of 1/B seconds. The difficulty is apparently due to variance in interpreting equation (2).

Reference 1 considers that equation (2) gives the entropy per mode while Reference 2 considers that equation (2) gives the entropy per degree of freedom (DOF). This leads to a conclusion by Reference 2 that the entropy rate is twice as great as given by equation (5).

Another difference is that Reference 2 stipulates that the wave bandwidth B must be small compared to the center frequency, while Reference 1 imposes no such restriction.

Both references make the following assumptions:

1. There is no useable information in the polarization or transverse distribution of intensity of the wave.

2. The information is entirely contained in the distribution of quanta among the modes or Nyquist intervals. In effect the receiver works on the time distribution of photons, making no distinction between photons of different energy.

The latter assumption deserves some thought. It is based on the idea that the amplitudes and phases of the modes can be determined by measuring the amplitude of BT samples. The receiver does this by counting the photons received in each sample. However the photon count is only proportional to the sample amplitude if all the photons have the same energy. In fact, as shown in Reference 2, the photons in the maximum entropy wave have energy distributions corresponding to the Bose-Einstein distribution.

According to the uncertainty principle,

$$\Delta E \Delta t = h/2\pi \quad \text{where} \tag{7}$$

ΔE is the uncertainty in the energy of the photon and Δt is the uncertainty in the time of arrival of the photon. The energy of the photon is hf. Then $\Delta E = h\Delta f$

Substituting in (7)

$$\Delta f = 1/2\pi\Delta t \tag{8}$$

This is approximately the same as the frequency resolution of a counting process in Δt seconds. It follows that measurement of photon energy would only supply redundant information to the measurement of photon count.

Having calculated the information capacity of a maximum entropy wave, the next step is to calculate the information capacity of a wave containing additive noise.

According to Shannon, the information capacity of such a wave is equal to the total entropy of wave less the entropy of the noise, providing that both signal and noise have the statistics of white noise.

In order to calculate the total entropy of the wave it is necessary that the distributions of quanta in modes for signal and noise be additive. At this point, Reference 1 treats the maximum entropy (exponential) distribution of equation (3) as if it were additive and calculated the total wave entropy by adding signal and noise having this distribution.

On the basis that the exponential distribution is non-additive, Reference 2 calculates channel capacity for signal and noise having Poisson distribution.

Reference 1 obtains,

$$C_w = B \log \left(1 + \frac{S}{N + hvB}\right) + \frac{S + N}{hv} \log \left(1 + \frac{hvB}{S + N}\right) \tag{9}$$

$$- \frac{N}{hv} \log \left(1 + \frac{hvB}{N}\right), \text{ where}$$

S = Signal power, N = Noise Power,
and claims validity for all ranges of mode occupation numbers (quanta per mode).

Reference 2 writes four equations for four different combinations of ranges of occupation numbers. The results are the same for large occupation numbers if allowance is made for the fact that Reference 2 assumes 2B rather than B measurements per second.

Equation (5) can be interpreted as representing the sum of two forms of entropy. The first term represents entropy in the form of multiple occupation numbers and predominates when $m \gg 1$. It is the form of information in multiple level coding such as analogue transmission. It is equal to the rate of mode arrival times the logarithm of the number of frequently occurring mode occupation numbers. The second term represents entropy in the form of pulse position rather than pulse amplitude and predominates when $m \ll 1$. It is the form of information in binary coding or PPM. It is equal to the rate of photon arrival times the logarithm of the number of frequently occurring modes per photon.

In equation (9), the first term corresponds to the multiple occupation form of signal entropy, the second term to the pulse position form of signal plus noise entropy, and the third term to the pulse position form of noise entropy.

The information capacity of a wave is plotted in Figure 1 as two components. The component labelled C_A is information corresponding to the first term of equation (9). The component labelled $C_B - C_C$ corresponds to the difference between the second and third terms of equation (9). CA increases with occupation number and finally approaches the classical limit for the given SNR. $C_B - C_C$ peaks at $\underline{m} = 5 \times 10^{-2}$ and falls off on either side. The decrease for large m is caused by excess mode occupation density. This decreases the number of modes available per photon.

If the relation $\bar{m} = \frac{S}{h\nu B}$ is substituted in equation (9),

$$C_W \to B \left[\log \left[1 + \bar{m}\left(1 + \frac{N}{S}\right) \right] + \bar{m}\left(1 + \frac{N}{S}\right) \log \left(1 + \frac{1}{\bar{m}\,(1 + N/S)}\right) \right.$$

$$\left. - \log \left(1 + \bar{m}\,\frac{N}{S}\right) - \bar{m}\,\frac{N}{S} \log \left(1 + \frac{S}{\bar{m}\,N}\right) \right] \qquad (10)$$

The information capacity of the wave is a function of $\bar{m}$, B, and S/N only.

Further, substituting $B = \frac{S}{h\nu\bar{m}}$,

$$\frac{C_W}{S} = \frac{1}{h\nu\bar{m}} \left[\log \left[1 + \bar{m}\left(1 + \frac{N}{S}\right)\right] + \bar{m}\left(1 + \frac{N}{S}\right) \log \left[1 + \frac{1}{\bar{m}\,(1 + N/S)}\right] \right.$$

$$\left. - \log \left(1 + \bar{m}\frac{N}{S}\right) - \bar{m}\frac{N}{S} \log \left(1 + \frac{S}{\bar{m}\,N}\right) \right] \qquad (11)$$

The information efficiency, $\frac{C_W}{S}$, of a wave is seen to be a function of $\bar{m}$ and S/N only.

Equation (11) is plotted as $\frac{C_W}{S}$ vs $\bar{m}$ with S/N as a parameter for a range of variables: (Figure 2)

$\bar{m}$ 10^{-4} to 10
S/N 10^{-3} to 10^{+3}
$h\nu = 3 \times 10^{-19}$ joules, 6800 Å.

The range of $\bar{m}$ is sufficient to demonstrate the transition from the classical case $m > 1$ to the quantum case $m < 1$. The information efficiency continues to rise without limit as m is decreased. However, the increase is very slow for $m\frac{N}{S} < 1$. The effect of additive noise is to decrease the information efficiency at all values of m, but for $S/N > m$ the effect of additive noise is very slight.

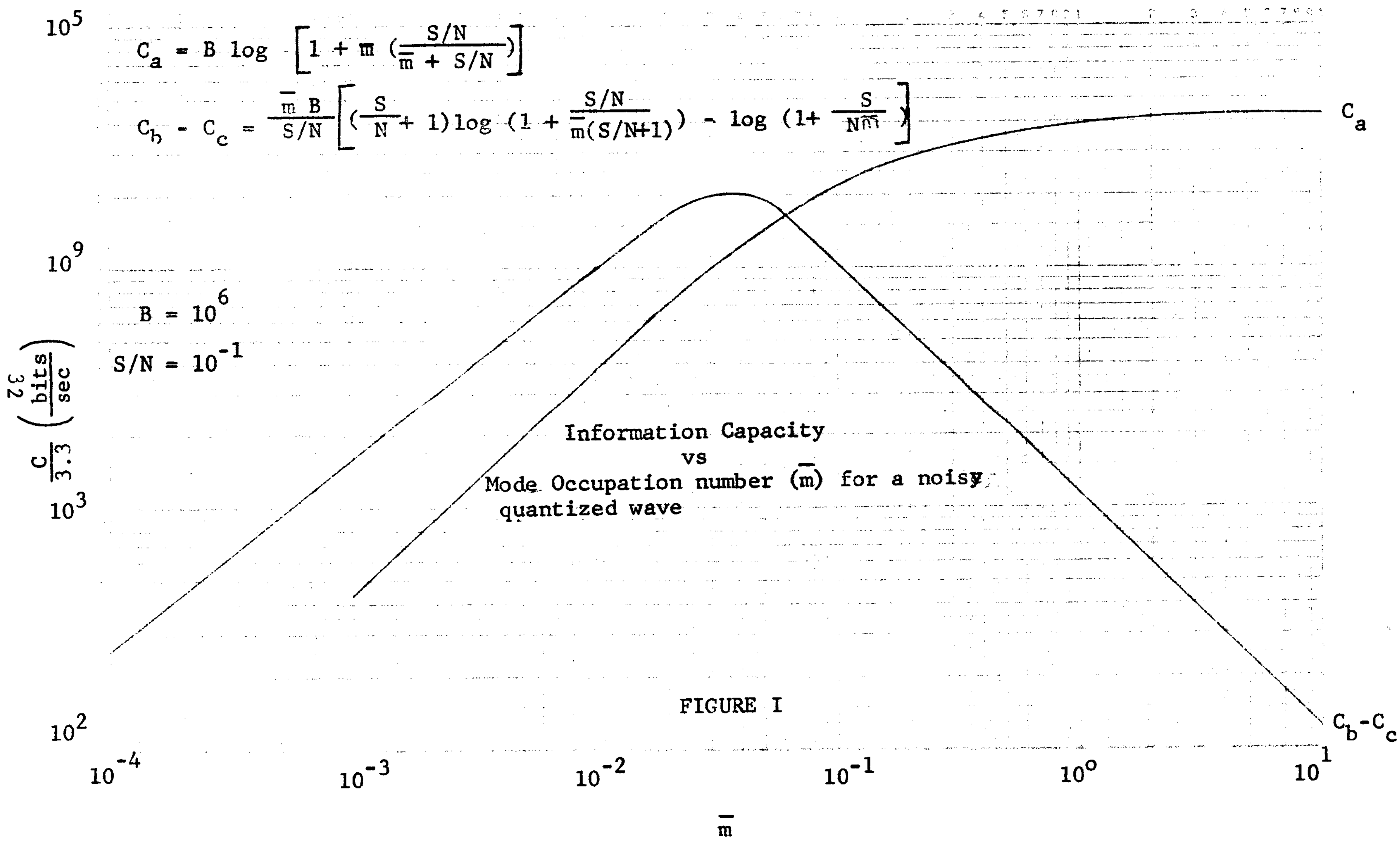

$C_a = B \log \left[1 + \bar{m} \left(\frac{S/N}{\bar{m} + S/N} \right) \right]$
$C_b - C_c = \frac{\bar{m}\, B}{S/N} \left[\left(\frac{S}{N} + 1 \right) \log \left(1 + \frac{S/N}{\bar{m}(S/N+1)} \right) - \log \left(1 + \frac{S}{N\bar{m}} \right) \right]$
$B = 10^6$
$S/N = 10^{-1}$
Information Capacity
vs
Mode Occupation number $(\bar{m})$ for a noisy
quantized wave
C_a
$C_b - C_c$
10^5
10^9
10^3
10^2
$\frac{C}{3.3}$ $\left(\frac{\text{bits}}{\text{sec}}\right)$
32
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^0
10^1
$\bar{m}$

FIGURE I

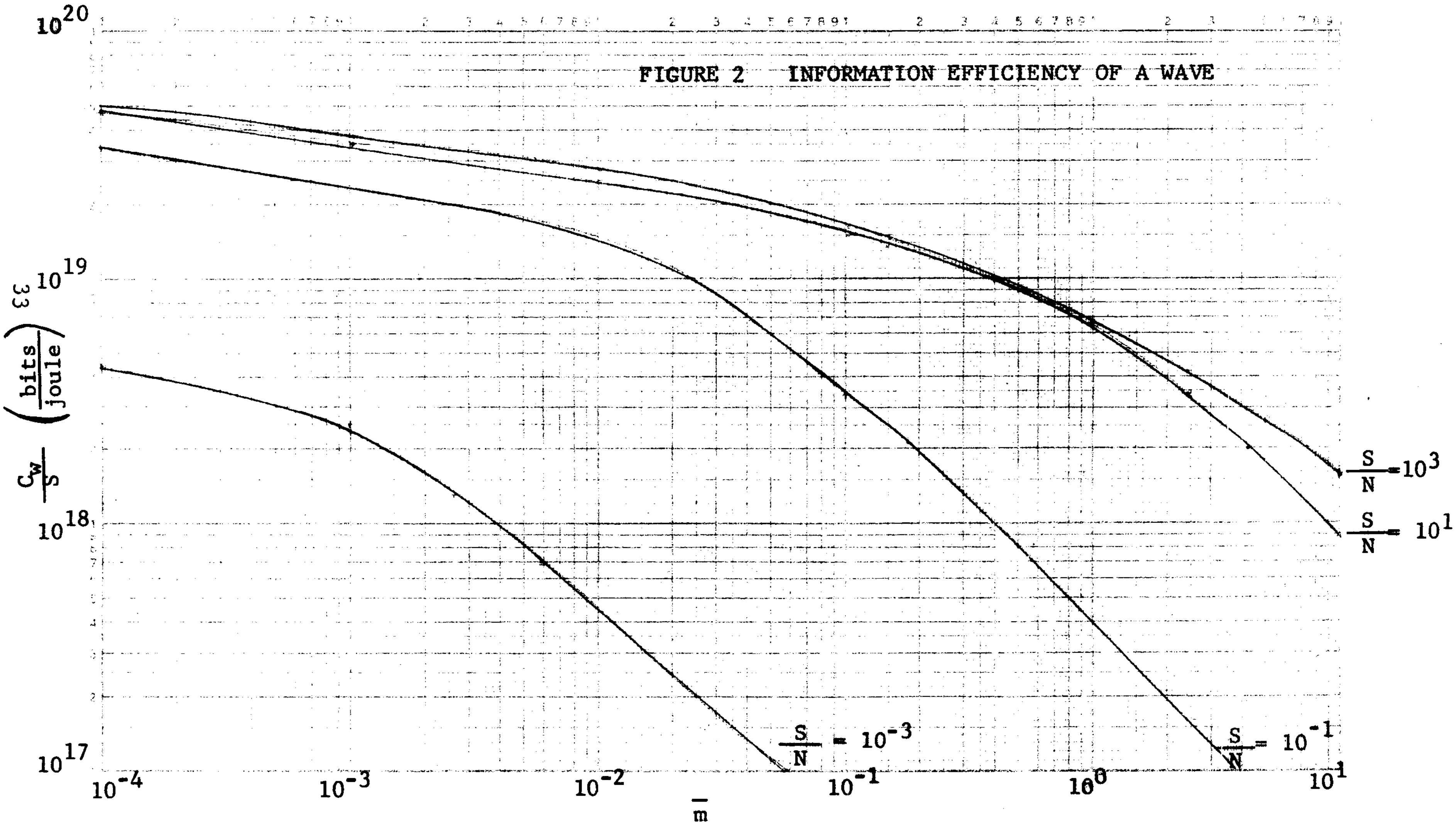


FIGURE 2 INFORMATION EFFICIENCY OF A WAVE

SECTION 9

RECEIVER PERFORMANCE CHARACTERISTICS

The information efficiency of heterodyne, homodyne, and quantum counter receivers will be calculated and compared to the previous curves which give the information efficiency of a receiver which is capable of extracting all of the information in the wave. The comparison will be on the basis of equal noise. This does not give a true performance comparison since the receivers have different capabilities for discriminating against noise outside the signal bandwidth. This distinction will be made later. A quantum efficiency of 10% is assumed for all receivers.

I. HETERODYNE RECEIVER

The heterodyne receiver mixes the signal wave with a locally generated coherent wave on the surface of a photo-detector. The resulting difference frequency electrical signal is processed and detected as in a radio receiver. The purpose and advantage is that there is a process gain for the signal and noise within the signal bandwidth, but not for the out-of-band incident noise nor the detector shot noise.

The following components of power are considered incident on the photosurface:

- P_S - The signal power
- P_{LO} - The local oscillator power in the main mode
- P_oB_o - The background noise spectral intensity as seen through an optical filter of bandwidth B_o
- $P_{NOB}B$ - The local oscillator noise power in the signal bandwidth B
- P_{NO} - The local oscillator noise power outisde the band B.

To these must be added an incident power P_{DC} equivalent to the dark current.

These incident power cause an average photocurrent

$$I_o = \frac{e}{hv} (P_S + P_{LO} + P_oB_o + P_{NOB}B + P_{NO} + P_{DC}) \tag{12}$$

This average photocurrent contains a shot noise component in the signal bandwidth B, the mean square value being

$$I_{SN}^2 = 2\ e\ BI_o = \frac{2\,\epsilon\, e^2 B}{h\ v}\ (P_S + P_{LO} + P_oB_o + P_{NOB}B + P_{NO} + P_{DC}) \qquad (13)$$

The local oscillator wave mixes with the incident power which lies in the signal bandwidth and produces a heterodyne component of photocurrent,

$$I_{het}^2 = 2\ \left(\frac{e}{hv}\right)^2\ P_{LO}\ (P_S + P_oB + P_{NOB}\ B) \qquad (14)$$

One component of I_{het}^2 is the mean square signal current.

$$I_S^2 = 2\ \left(\frac{e}{hv}\right)^2\ P_{LO}\ P_S \qquad (15)$$

The mean square noise current consists of the remainder of I_{het}^2 plus the shot noise

$$I_N^2 = 2\ \left(\frac{\epsilon e}{hv}\right)^2\ P_{LO}\ (\ P_oB + P_{NOB}B)$$

$$\frac{2\,\epsilon\, e^2\ B}{h\ v}\ (P_S + P_{LO} + P_o\ B_o + P_{NOB}B + P_{NO} + P_{DC}) \qquad (16)$$

Self beating between the components of incident power is accounted for in the shot noise term. The power SNR in the IF amplifier is,

$$\frac{I_S^2}{I_N^2} = \frac{\epsilon}{h\ v\ b}\left[\frac{P_{LO}P_S}{P_S + P_{LO} + P_oB_o + P_{NOB}B + P_{NO} + P_{DC} + \frac{\epsilon}{hv}\ P_{LO}(P_o + P_{NOB})}\right]$$

$$(17)$$

$$\text{If } P_{LO} \gg \frac{P_S + P_oB_o + P_{NOB}B + P_{NO} + P_{DC}}{1 + \frac{\epsilon}{h\ v}\ (P_o + P_{NOB})}$$

$$SNR_{IF} = \frac{\epsilon}{hvB}\left[\frac{P_S}{1 + \frac{\epsilon}{hv}\ (P_o + P_{NOB})}\right] \qquad (18)$$

From equation (15), the power gain of the heterodyne process is

$$\frac{I_S^2}{P_S} = 2\left[\frac{\epsilon e}{hv}\right]^2 P_{LO} \tag{19}$$

The noise power referred to the input is (16)

$$\frac{I_N^2}{2\left[\frac{\epsilon e}{hv}\right]^2 P_{LO}} = P_o B + P_{NOB}B + \frac{hvB}{\epsilon}\left[(1 + P_S + P_o B_o + P_{NOB}B + P_{NO} + P_{DC})\right] \tag{20}$$

The noise power referred to the input is greater than hvB even with no incident noise. It follows that the information contained in the last two terms of equation (9), in the form of pulse position rather than amplitude, is lost in the heterodyne process.

The information capacity of the heterodyne receiver is therefore obtained by the first term of equation (9) with $N > h\,v\,B$.

$$C_h = B \log\left(1 + \frac{S}{N}\right) = B \log\left[1 + \frac{\epsilon}{hvB}\left(\frac{P_S}{1 + \epsilon/hv\,(P_o + P_{NOB})}\right)\right] \tag{21}$$

The information efficiency of the heterodyne is

$$\frac{C_h}{P_S} = \frac{B}{P_S} \log\left[1 + \frac{\epsilon}{hvB}\left(\frac{P_S}{1 + \epsilon/hv\,(P_o + P_{NOB})}\right)\right]$$

Equation (9) was derived on the basis of B being the information bandwidth of the wave. But in equations (12-31), B is taken as the bandwidth of the IF channel. It follows that equations (21) and (22) require that 2B samples be taken per second. This is permissible for large m but not for small m.

Equation (22) is plotted so that all of the incident noise is in the signal bandwidth. This permits a direct comparison to the information efficiency of an incident noisy wave, C_W, but does not illustrate the advantage of the heterodyne in rejecting out-of-band incident noise. (Figure 3)

II. HOMODYNE RECEIVER

In the homodyne receiver, the local oscillator laser is locked to the average phase of the received carrier. If the carrier is biphase modulated with a phase shift of $\pm\pi/2$ radians, the resulting mean square signal current is

$$I_S^2 = 4 \left[\frac{\epsilon e}{h\nu}\right]^2 P_{LO} P_S \tag{23}$$

This is twice the heterodyne mean square signal current because the mixing signals are always exactly in phase or exactly out of phase. Furthermore, the post detection filter bandwidth is only B/2 instead of B as in the heterodyne IF. This reduces the shot noise to 1/2 of the value given in equation (13). The noise component of $I^2{}_{het}$, equation (14) is unchanged.

$$I_N^2 = 2 \left(\frac{\epsilon e}{h\nu}\right)^2 P_{LO}\left[P_o B + P_{NOB}B\right] \tag{24}$$

$$+ \frac{e^2 B}{h\nu} \left(P_S + P_{LO} + P_o B_o + P_{NOB}B + P_{NO} + P_{DC}\right)$$

The noise power referred to the input is usually greater than $h\nu B$. Therefore the information capacity is given by substitution in the first term of equation (9), with the information bandwidth equal to B/2.

$$\frac{C_{ho}}{P_S} = \frac{B}{2P_S} \log\left[1 + \frac{\epsilon}{h\nu B}\left(\frac{4\, P_S}{1 + \frac{2\epsilon}{h\nu}\left(P_o + P_{NOB}\right)}\right)\right] \tag{25}$$

The homodyne receiver information efficiency is plotted on the same basis as the heterodyne receiver. The results are similar except for an improvement of two to one at low values of m. (Figure 4)

III. QUANTUM COUNTER RECEIVER

The quantum counter receiver consists merely of a photo detector coupled to a device which counts photo-electrons with a time resolution equal to 1/B. This process is exactly equivalent to measuring the amplitude of the wave modes and should therefore extract all of the information in the wave as defined by Reference 1. This is found to be the case for $S \ll h \nu B$ in a noise free channel.

For high occupation numbers the quantum counter can extract less than half of the wave information. This is rather surprising in view of the assumptions made in calculating the wave capacity. It is explained by Reference 1 by the assertion that an energy sensitive receiver cannot extract information in the form of phase. However, his calculation of wave capacity excluded the possibility of amplitude and phase measurement. These questions deserve further consideration and will be treated in a following section. It is probable that the present results will not be greatly affected.

There is no single formula available for calculating the information efficiency of a quantum counter receiver over the whole range of occupation numbers. For low occupation numbers, it is assumed that the receiver distinguished only two states: a zero and a one. The probability of multiple occupation is so low that little information is lost by this procedure. The quantum counter capacity is calculated by assuming a Poisson distribution in the number of photons received from a constant amplitude transmitted pulse.

For high occupation numbers it is assumed that multiple level coding is used. The quantum counter capacity is calculated by a similar process.

For occupation numbers near unity the extremes are joined by a smooth curve. Reference 1 is followed in the above work. The curves of information efficiency are plotted for a quantum efficiency of 10%. (Figure 5).

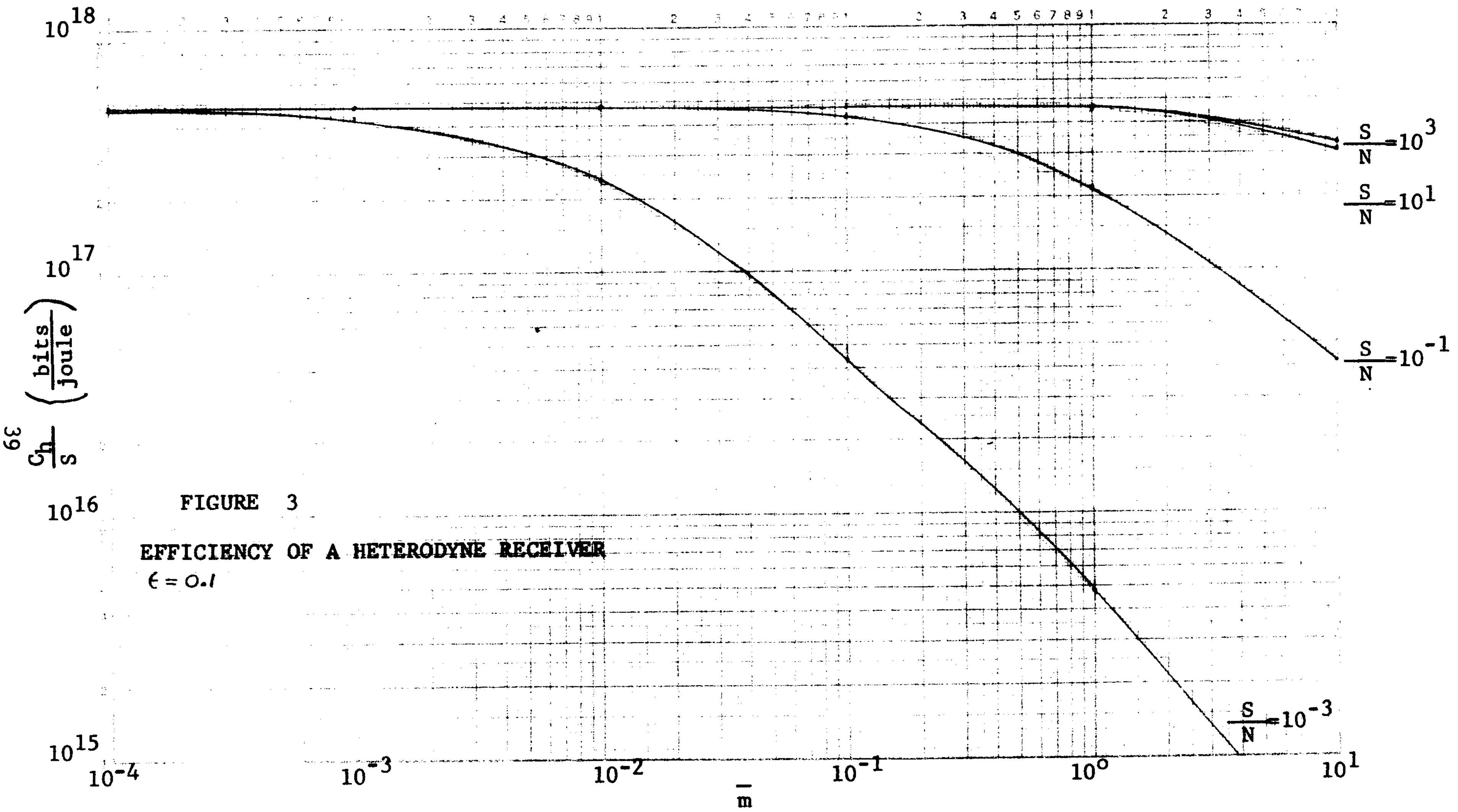


FIGURE 3

EFFICIENCY OF A HETERODYNE RECEIVER

$\epsilon = 0.1$

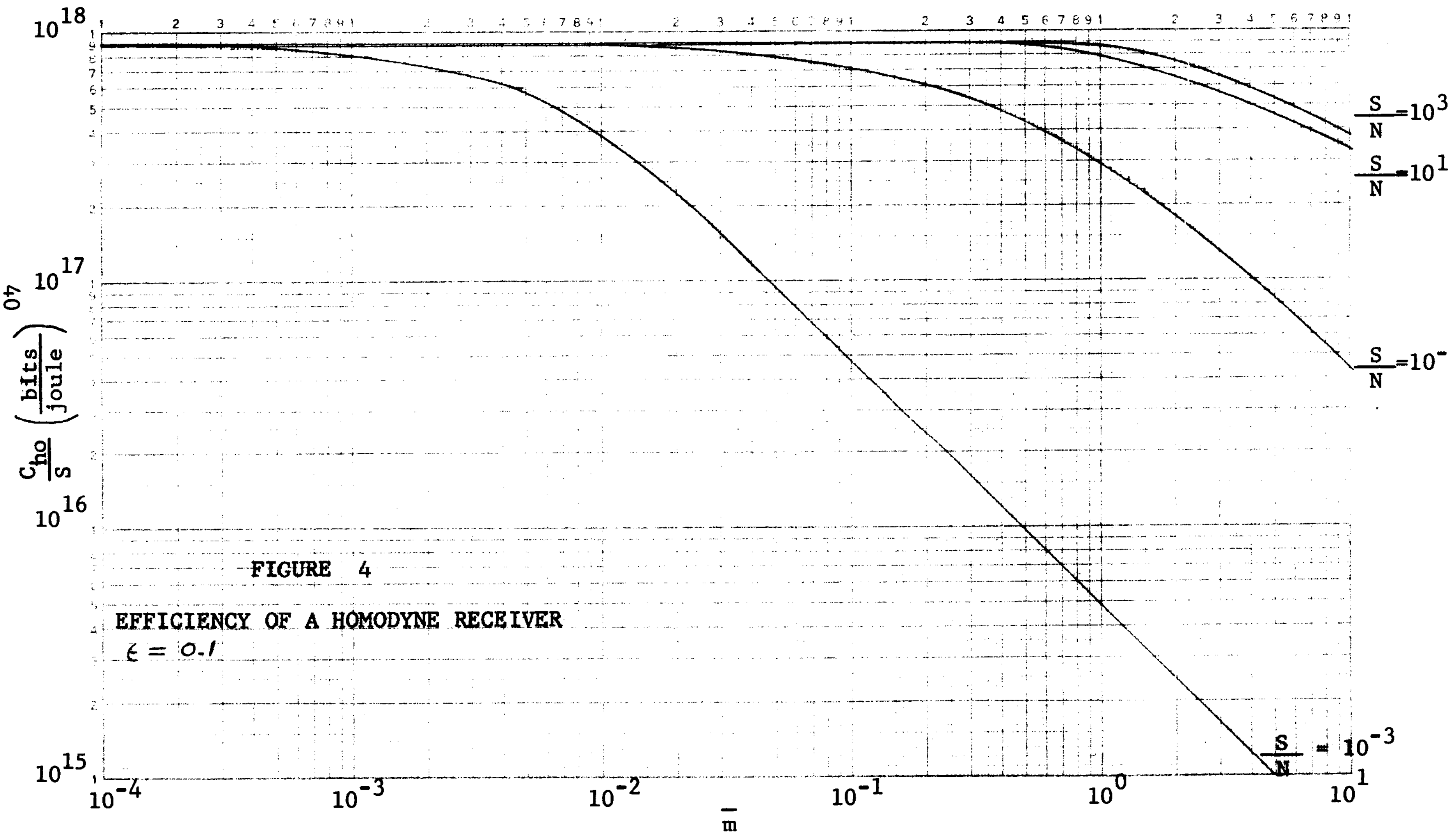


FIGURE 4

EFFICIENCY OF A HOMODYNE RECEIVER

$\epsilon = 0.1$

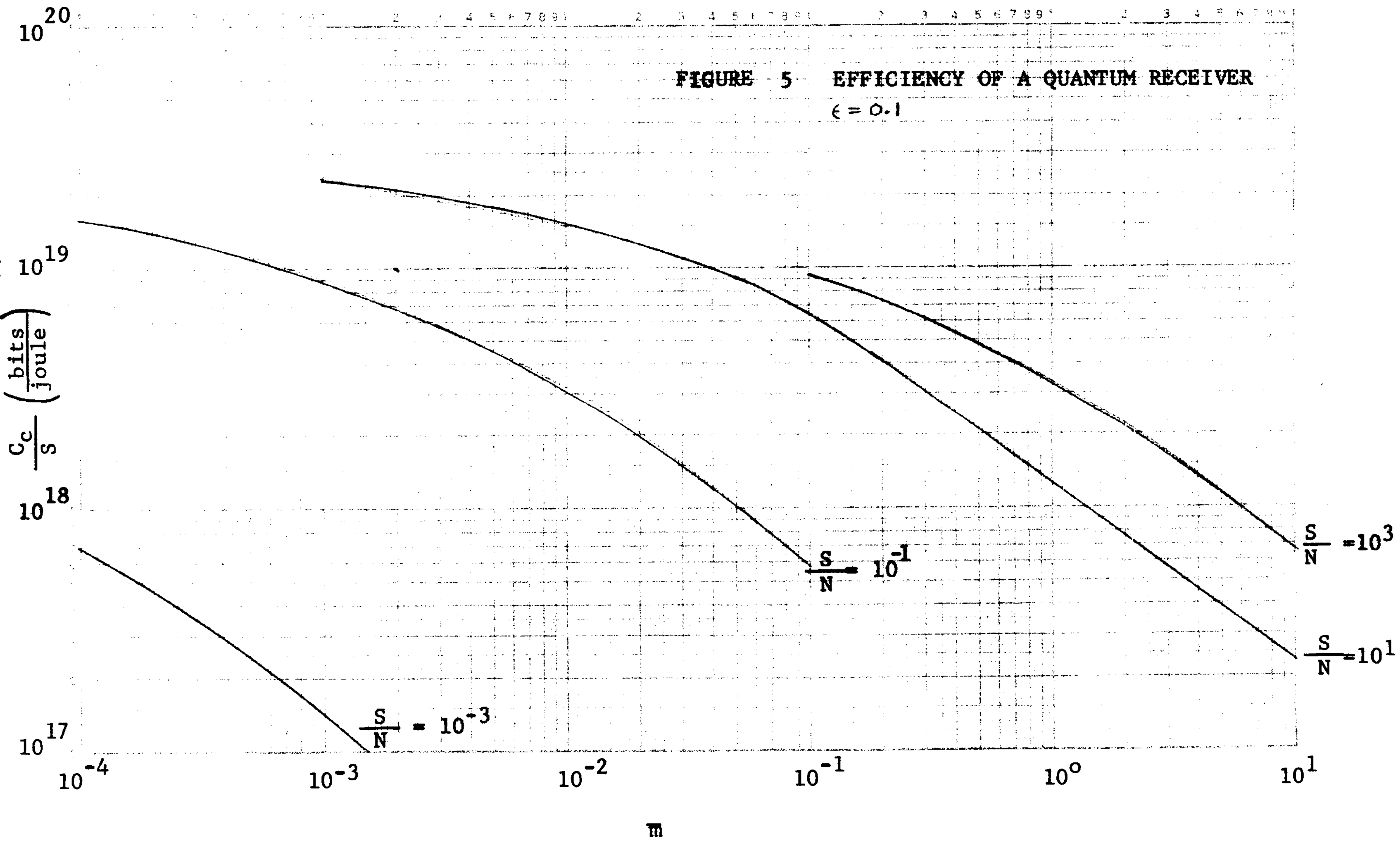


FIGURE 5 EFFICIENCY OF A QUANTUM RECEIVER
$\epsilon = 0.1$

SECTION 10

INTERPRETATION OF RESULTS

I. WAVE INFORMATION EFFICIENCY

The information efficiency of a wave has been calculated for a wide range of $\bar{m}$ and SNR. This form of presentation eliminates bandwidth as a factor in the wave information efficiency. If the limitations of the receiver are ignored, certain significant generalizations are apparent:

1. Efficiency increases as $\bar{m}$ decreases.

2. For the range of $\bar{m}$ investigated (10^{-4} - 10^{1}) there is no significant improvement in information efficiency for SNR $>$ 10.

The interpretation of these results must be strictly confined to the case of an ideal receiver which extracts all of the wave information. For this case, it seems that maximum efficiency will be obtained with a system having wide bandwidth (this implies small $\bar{m}$) and having some form of binary coding of PPM. It is of interest to observe that the information efficiency of the wave at $\bar{m} = 10^{-4}$ is 4.7×10^{19} bits per joule. This corresponds to $4.7 \times 10^{19} \times 3 \times 10^{-19} = 14$ bits per photon. The capacity of a PPM system with 10^4 spaces between pulses is $4 \times 3.3 = 13$ bits per pulse. This implies that a PPM system which receives 1 photon per pulse would extract essentially all of the information in a wave of low occupation number.

II RECEIVER INFORMATION EFFICIENCY

The performances of heterodyne, homodyne, and quantum counter receivers have been calculated and compared to the performance of a hypothetical perfect receiver in extracting information from an optical wave. The receivers differ significantly in performance under various conditions of mode occupation number and SNR. The basis for comparison is relative information efficiency under identical conditions of mode occupation number, noise power <u>in the signal bandwidth</u>, and quantum efficiency. Maximum entropy coding is assumed in all cases.

A. <u>HETERODYNE RECEIVER</u>

It was shown that the heterodyne and homodyne receivers respond only to information of the type represented by C_A in Figure 1. This is because the local oscillator produces shot noise in excess of $h\nu B$ (See equation 9). Because of this limitation, the heterodyne receiver cannot operate in the low $\bar{m}$ regime which was shown to be the most efficient form of information in a wave. At high occupation numbers, the heterodyne receiver surpasses the efficiency of the quantum counter on the basis of equal noise in the signal bandwidth without accounting for out of band noise.

The reason for the superiority of heterodyne detection at high occupation numbers is that 2B samples are taken per second, twice as much as allowed for the quantum counter receiver. This is justified on the basis that the heterodyne receiver is sensitive to phase as well as energy. There is still some question about the correctness of this viewpoint. However, the real superiority of heterodyne detection undoubtedly lies in the rejection of out of band incident noise.

An inspection of the heterodyne receiver efficiency curves shows that there is no improvement in information efficiency as S/N is increased beyond $\bar{m}\,\epsilon$. This makes it possible to specify some of the requirements for an optical collecting system.

Let $\frac{S}{N} = \bar{m}\,\epsilon$ and substitute $\frac{S}{h\nu B} = \bar{m}$,

$$\frac{S}{N} = \frac{S\,\epsilon}{h\nu B}$$. From which

$$N = \frac{h\nu B}{\epsilon},$$

Also $N = P_o B$, where P_o is the incident noise spectral intensity.

Then,

$$P_o B = \frac{h\nu B}{\epsilon}, \text{ or}$$

$$P_o = \frac{h\nu}{\epsilon} = \frac{3 \times 10^{-19}}{\epsilon} \text{ joules or watts - cps}^{-1} \tag{26}$$

This defines a level of noise spectral density beyond which information efficiency is sacrificed.
Let the incident noise spectral radiance be

I_o watts cycle^{-1} M^{-2} st^{-1}

Then

$P_o = I_o \left(\frac{\pi}{4}\right)^2 D_o^2\, \alpha_o^2$, where D_o and α_o are the diameter (27)

and field-of-view of the primary collecting optics.

From (26) and (27),

$$D_o^2 \; \alpha_o^2 = \frac{hv}{\epsilon I_o \left(\frac{\pi}{4}\right)^2} \qquad (28)$$

Equation (28) sets an upper limit to the useful quality of the collecting optics. It will be of interest to compare this limit to the limit imposed by wave congruence. Let,

$I_o = 4.7 \times 10^{-15}$ W - M^{-2} st^{-1} - cps^{-1}, a typical value for day sky.

Substituting this value with $\epsilon = 0.1$ and $hv = 3 \times 10^{-19}$,

$$D_o^2 \; \alpha_o^2 = \frac{3 \times 10^{-19}}{10^{-1} \times 4.7 \times 10^{-15} \left(\frac{\pi}{4}\right)^2} = 10^{-3}$$

$$D_o \; \alpha_o = 3.3 \times 10^{-2} \text{ M} \qquad (29)$$

The wave congruence requirement is,

$D_o \; \alpha_o = 4.4 \times 10^{-7}$ M for 50% loss of signal.

If the wave congruence requirement could be relieved in some way, it would be possible to use large, low quality collecting optics. For example, equation (29) specified an acuity of 3.3 milliradians in a 10 M aperture with day sky background.

B. QUANTUM COUNTER RECEIVER

The quantum counter receiver must operate at a higher SNR than the heterodyne receiver for an equivalent information efficiency. Comparing figures 3 and 5, the quantum counter SN must be at least 10 $\bar{m}$ for a performance equivalent to the heterodyne receiver. The corresponding heterodyne receiver SNR is $\frac{\bar{m}}{10}$. This ratio is valid for quantum efficiencies in the neighborhood of 0.1. The quantum receiver accepts incident noise in the optical filter bandwidth B_o.

$$\frac{S}{N} = 10 \; \bar{m}$$

$$\frac{S}{P_o B_o} = 10 \; \bar{m}$$

Substituting the relation $S = \bar{m}$ hvB

$$\frac{\bar{m}\, h\, v\, B}{P_o B_o} = 10\, \bar{m}$$

Finally

$$\frac{h\, v\, B}{P_o B_o} = 10 \qquad (30)$$

But for the heterodyne receiver, the noise spectral density can be as high as

$$P_o = \frac{h\, v}{\epsilon} = 10\, h\, v \qquad \text{equation (26)}$$

Substituting in (30)

$$\frac{B}{B_o} = 100$$

It is of course impossible for the signal bandwidth B to exceed the optical filter bandwidth B_o. Even if the signal bandwidth were made equal to the optical filter bandwidth by extreme wideband modulation, the resulting noise discrimination would still be inferior to the heterodyne receiver by a factor of 100. For example, in the case of day sky background cited previously, the value of $D_o \alpha_o$ would be 1/10 as large as given by equation (29) for the heterodyne receiver.

$(D_o \alpha_o = 3.3 \times 10^{-3} M)$.

Note that this limit is still far less exacting than the requirement for heterodyne wave congruence.

$(D_o \alpha_o = 4.4 \times 10^{-7}\ M)$

Taking the latter value, we can calculate the signal bandwidth required for equivalent performance in a quantum counter receiver. Assume $B_o = 1\ \text{Å} = 6.32 \times 10^{10}$ cps.

From equation (30)

$$B = \frac{10\, P_o B_o}{h\, v} \qquad (32)$$

Substitute $D_o \alpha_o = 4.4 \times 10^{-7}$ and

$I_o = 4.7 \times 10^{-15}$ in equation (27), and solve for p_o.

Substitute in equation (32)

$$B = \frac{10 \times 6.32 \times 10^{16}}{3 \times 10^{-19}} (4.7 \times 10^{-15}) \ (\pi/4) \ (4.4 \times 10^{-7})$$
$$= 1500 \text{ cps} \tag{33}$$

This is a very easily met requirement.

We interpret equation (33) as follows:

Given day sky background and an optical system of quality consistent with wave congruence, the quantum counter receiver will perform as well as the heterodyne receiver, assuming the availability of 1 Å optical filters.

SECTION 11

PHOTOMIXING WITH NON-CONGRUENT WAVES

If the signal and local oscillator waves are not everywhere parallel on the photosurface, the components of IF photocurrent will not be in phase and the total IF photocurrent will be partially cancelled.

The photosurface is uniformly illuminated by a local source of frequency w_o and by a signal w_s with amplitude modulation w_m. At an element of photosurface dy, the sum amplitude is,

$$E_s \left[1 + \sin w_m (t-\tau) \right] \quad \sin w_s (t-\tau) + E_o \sin w_o t \qquad (1)$$

where

E_s is the signal carrier amplitude
E_o is the local source amplitude
τ is the delay between signal and local source zero crossings at a point y on the photosurface.

The photocurrent from the element dy has a component di which contains the difference frequencies:

$$di = K_1 E_s E_o \cos \left[(w_s - w_o)t - w_s \tau \right] dy \qquad (2)$$

$$- \frac{K_1 E_s E_o \sin}{2} \left[(w_s - w_o - w_m)t - \tau \ (w_s - w_m) \right] dy$$

$$+ \frac{K_1 E_s E_o \sin}{2} \left[(w_s - w_o + w_m)t - \tau \ (w_s + w_m) \right] dy$$

where

K_1 is a gain constant.

In order to determine the total difference frequency components of the photocurrent, the elemental currents di must be integrated over the length Y.

If the signal and local source waves are plane and at an angle α with respect to each other, the delay τ becomes a function of Y

$$\tau = \frac{\alpha Y}{c}, \text{ where c is the velocity of light} \qquad (3)$$

and τ is zero at the origin.

The sum amplitude becomes (1)

$$E_s\left[1 + \sin w_m \left(t - \frac{\alpha Y}{c}\right)\right] \quad \sin w_s \left(t - \frac{\alpha Y}{c}\right) + E_o \sin w_o t \tag{4}$$

The element of difference frequency photocurrent at Y is (2)

$$di = K_1 E_s E_o \cos\left[(w_s - w_o)\, t - w_s \frac{\alpha Y}{c}\right] dy \tag{5}$$

$$- \frac{k_1 E_s E_o}{2} \sin\left[(w_s - w_o - w_m)t - \frac{\alpha Y}{c}(w_s - w_m)\right] dy$$

$$+ \frac{K_1 E_s E_o}{2} \sin\left[(w_s - w_o + w_m)t - \frac{\alpha Y}{c}(w_s + w_m)\right] dy$$

The total difference frequency photocurrent

$$I = \int_0^Y di \tag{6}$$

$$= -K_1 E_s E_o \left(\frac{c}{w_s \alpha}\right) \sin\left[(w_s - w_o)t - w_s \frac{\alpha y}{c}\right]$$

$$- \frac{K_1 E_s E_o}{2}\left(\frac{c}{\alpha (w_s - w_m)}\right) \cos\left[(w_s - w_o - w_m)t = \frac{\alpha y}{c}(w_s - w_m)\right]$$

$$+ \frac{K_1 E_s E_o}{2}\left(\frac{c}{\alpha (w_s + w_m)}\right) \cos\left[(w_s - w_o + w_m)t - \frac{\alpha y}{c}(w_s + w_m)\right]\Bigg|_0^Y$$

$$I = -K_1 E_s E_o \left(\frac{c}{w_s \alpha}\right)\left[\sin\left[(w_s - w_o)t - \frac{w_s \alpha Y}{c}\right] - \sin\left[(w_s - w_o)t\right]\right]$$

$$- \frac{K_1 E_s E_o}{2}\left(\frac{c}{\alpha (w_s - w_m)}\right)\left[\cos\left[(w_s - w_o - w_m)t - \frac{\alpha Y}{c}(w_s - w_m)\right] - \cos (w_s - w_o - w_m)t\right]$$

$$+ \frac{K_1 E_s E_o}{2}\left(\frac{c}{\alpha (w_s + w_m)}\right)\left[\cos\left[(w_s - w_o + w_m)t - \frac{\alpha Y}{c}(w_s + w_m)\right] - \cos (w_s - w_o + w_m)\, t\right]$$

$$= K_1 E_s E_o \frac{c}{\alpha}\left[\frac{-2}{w_s} \sin\left(\frac{w_s \alpha Y}{2c}\right) \cos\left[(w_s - w_o)t - \frac{w_s \alpha Y}{2c}\right]\right.$$

$$+ \frac{1}{w_s - w_m} \sin\left(\frac{-\alpha Y (w_s - w_m)}{2c}\right) \sin\left[(w_s - w_o - w_m)t - \frac{\alpha Y (w_s - w_m)}{2c}\right]$$

$$\left. - \frac{1}{w_s + w_m} \sin\left(\frac{-\alpha Y (w_s + w_m)}{2c}\right) \sin\left[(w_s - w_o + w_m)t - \frac{\alpha Y (w_s + w_m)}{2c}\right]\right]$$

If the total difference frequency current is detected in a square law detector, the output modulation component is:

$$I_m = \left[K_1 E_s E_o \frac{c}{\alpha} \right]^2 \times \tag{7}$$

$$\left\{ + \frac{2}{w_s(W_s - w_m)} \sin\left(\frac{-w_s \alpha Y}{2c}\right) \sin\left(\frac{-\alpha Y(w_s - w_m)}{2c}\right) \sin\left[+w_m t - \alpha \frac{Y w_m}{2c}\right]\right.$$

$$\left. + \frac{2}{w_s(w_s + w_m)} \sin\left(\frac{-w_s \alpha Y}{2c}\right) \sin\left(\frac{-\alpha Y(w_s + w_m)}{2c}\right) \sin\left[w_m t + \frac{\alpha Y}{2c} wm\right]\right\}$$

When $w_m << w_s$ and $\frac{\alpha Y}{2c} w_m$ is a very small angle

$$I_m = 4 \left[\frac{K_1 E_s E_o c}{w_s \alpha}\right]^2 \sin^2 \frac{w_s \alpha Y}{2c} \sin\left[(w_m t)\right] \tag{8}$$

Equation (8) gives the modulation current for the case where the signal and local source waves are plane, and have an angle α between them. Since the order of summation of di is immaterial, equation (8) is also valid for a random distribution of delays across Y, where any delay from 0 to $T = \frac{\alpha Y}{c}$ is equally likely.

For distribution of delay across Y other than those mentioned, equation (3) must be modified to the appropriate function of Y.

From equation (8) if I_m is the demodulated signal for perfect wave congruence and I_m^1 is the signal for a misalignment angle α,

$$I_m^1 = I_m \left[\frac{2c}{w_s Y \alpha}\right]^2 \sin^2 \frac{w_s \alpha Y}{2c} \tag{9}$$

When $I_m^1 / I_m = 0.5$,

$$\frac{w_s \alpha Y}{2c} = 1.4$$

For a typical w_s of 1.9×10^{15} rad sec^{-1} (λ = 1 micron)

$$Y\alpha = \frac{1.4 \times 2 \times 3 \times 10^8}{1.9 \times 10^{15}} = 4.4 \times 10^{-7} \text{ meters} \tag{10}$$

$$= .44\ \lambda$$

The first zero occurs when

$$Y\alpha = \frac{2\pi c}{w_s} = \frac{2\pi \times 3 \times 10^8}{1.9 \times 10^{15}} = 10^{-6} \text{ meters} \qquad (11)$$
$$= \lambda$$

Equation (9) shows that the conversion gain becomes zero when α is an integer multiple of 2 $c/w_s Y$. This corresponds to a maximum displacement of an integer number of wavelengths across Y.

The foregoing analysis considered the effect of the mixing wavefronts being non-congruent. The position of the photosurface with respect to the incicent light will now be considered. If the photosurface is tilted with respect to the congruent mixing waves an angle α, the resulting element of photocurrent at Y is,

$$di = K_1 E_s E_o \left[\begin{array}{l} \cos\left[(w_s - w_o)t - (w_s - w_o)\alpha Y/c\right] \\ - \sin\left[(w_s - w_o - w_m)t - (w_s - w_o - w_m)\alpha Y/c\right] \\ + \sin\left[(w_s - w_o + w_m)t - (w_s - w_o + w_m)\alpha Y/c\right] \end{array} \right] dy \qquad (12)$$

Equation (12) differs from equation (5) only in that the delay $\alpha Y/c$ is acting on the difference frequency $(w_s - w_o)$ rather than the signal frequency w_s. Integrating (12) and extracting the modulation component of the detected photocurrent,

$$I_m = \frac{2}{w_s - w_o}\left[K_1 E_s E_o \frac{c}{\alpha}\right]^2 \sin\left[\frac{(w_s - w_o)\alpha Y}{2c}\right] \left[\begin{array}{l} \frac{1}{(w_s - w_o - w_m)} \sin\frac{(w_s - w_o - w_m)\alpha Y}{2c} \\ + \frac{1}{(w_s - w_o + w_m)} \sin\frac{(w_s - w_o + w_m)\alpha Y}{2c} \end{array} \right] \sin w\, t \qquad (13)$$

When $w_m < \dfrac{w_s - w_o}{10}$

$$I_m \simeq 4\left[\frac{K_1 E_s E_o c}{(w_s - w_o)\alpha}\right]^2 \sin^2\left[\frac{(w_s - w_o)\alpha Y}{2c}\right] \sin w\, t \qquad (14)$$

If I_m is the demodulated signal from a congruent photocathode, and I_m^1 is the signal for a misalignment angle α (or a surface roughness $Y\alpha$),

$$I_m^{1} = I_m \left[\frac{2c}{(w_s - w_o) Y\alpha} \right]^2 \sin^2 \frac{(w_s - w_o)}{2c} Y\alpha \qquad (15)$$

When $I_m^{1}/I_m = 0.5$,

$$\frac{(w_s - w_o)Y\alpha}{2c} = 1.4$$

If $(w_s - w_o) = 2\pi \times 10^9$ rad sec^{-1}

$$Y\alpha = \frac{1.4 \times 2 \times 3 \times 10^8}{2\pi \times 10^9} = 0.134 \text{ meters} \qquad (16)$$

which shows that photosurface alignment or surface roughness is not a problem.

Equations (9), (10) and (11) show how the conversion gain is affected by misalignment of the mixing wavefronts for plane or randomly distributed waves. In a practical system, the wave misalignment may be caused by

1. Imperfections in the collecting optics

2. Atmospheric refraction

3. Aiming error

The effect on the system performance will depend on whether the misalignment fluctuates or is constant. Atmospheric refraction and aiming error will cause the conversion gain to fluctuate. This amounts to a source of multiplicative noise or fast fading. It can be partially compensated by the use of AGC based on the strength of the detected IF carrier. However, if the misalignment becomes as large as indicated in equation (11) the conversion gain will become zero and AGC will not be effective.

If the misalignment is constant, say due to errors in a rigid mirror, the conversion gain will be low but constant. In these circumstances there will be no multiplicative noise, but the output SNR may be lowered. This is because certain sources of noise, such as local oscillator shot noise and dark current noise are independent of the mixer conversion gain. These effects will be analyzed in a following section.

Equation (10) shows that a misalignment equivalent to $Y\alpha = .44$ will result in a conversion loss factor of 0.5. The implications of this criterion on the optics will be discussed. Assume that the signal illumination is focussed on the photosurface. The dimension Y must be as large as the focused image if all of the signal power is to be used. In the case of a diffraction limited system, the central disc will have a diameter 1.2 (NA), where NA is the numberical aperture.

If the local source is also diffraction limited, its diffraction pattern can be focussed on the photosurface through a system having the same numerical aperture and the two diffraction patterns will be everywhere congruent in phase and angle of incidence. Therefore, the diffraction limited acuity of the system is not a factor in the conversion gain.

If the collector system is looking through the atmosphere, the image of the source will be influenced by atmospheric refraction, causing motion, distortion and defocussing of the image. Any resulting misalignment or curvature of the incident wave will be magnified at the photosurface by the ratio of the entrance aperture to the photosurface aperture. If the misalignment or curvature of the incident wave due to refraction is taken as 2 arc seconds (10^{-5} rad), the product $Y\alpha$ at the photosurface will be equal to $10^{-5}D_o$, where D_o is the entrance aperture of the collecting system. By the criterion of equation (10),

$$D_o \leqq 4.4 \text{ cm} \tag{17}$$

This limitation can be overcome by dividing the required large collecting area into a number of independent sub-collectors each of which meets the criterion of equation (10). Each sub-collector would be provided with a photodetector and the detector outputs would be summed.

Suppose that a collecting area of 10 M^2 is required. Then under equation (10) the number of sub-collectors in the array will be

$$n_c = \frac{10 \times 4}{\pi (4.4 \times 10^{-2})^2} = 6600 \tag{18}$$

In order to sum the detector outputs without self-cancellation, two methods are conceivable.

In principle, the difference frequency carrier and sideband components from each detector could be phase corrected prior to demodulation. This procedure is difficult because of the large number of elements involved. Secondly, the difference frequency components could be demodulated to base band before summation. Equation (7) shows that the phase dispersion in the demodulated signal is negligible; therefore, the base band signals can be summed without self-cancellation.

SECTION 12

EFFECT OF NON-CONGRUENCY ON S/N

The power SNR at the output of a coherent detector is,

$$(S/N)_P = \frac{2\,P^2\,P_r P_s}{i^2_d + i^2_b + i^2_c + i^2_{nr} + i^2_m} \quad (1)$$

where

$$i^2_d = 2\,e\,I_d\,\Delta f_m$$

$$i^2_b = 2\,e\,P\,P_b\,\Delta f_m$$

$$i^2_c = 2\,e\,P\,(P_s + P_r)\,\Delta f_m$$

$$i^2_{nr} = 2\,e\,P\,P_{NOB}\,\Delta f_m$$

I_d = dark current

P_b = background noise power

P_s = signal power

P_r = L. O. Power

P_{NOB} = L. O. Noise power

Δf_m = modulating frequency bandwidth

It will be assumed that the local oscillator noise bandwidth is small compared to the I.F. bandwidth. Also the continuous background noise spectrum within an optical filter bandwidth Δf_o will be approximated by discrete components separated by the I.F. (Δf_i). The magnitude of each component is

$$P_{bi} = P_b \left(\frac{\Delta f_i}{\Delta f_o} \right) \tag{2}$$

Mixing will occur for two sets of components:

1. Local oscillator and background noise beats

2. Signal and background noise beats

The photo-current due to (1) may be expressed as,

$$I_1 = 2 P \sqrt{P_r P_{bi}} \quad \cos(w_o - w_{bi})t \tag{3}$$

The mean square current is

$$i^2_{m1} = 2 P^2 P_r P_{bi} \tag{4}$$

Since mixing occurs for a noise component above and below the oscillator the total mean square current is

$$i^2_{m1} = 4 P^2 P_r P_{bi} \tag{5}$$

The mean square current due to signal and background noise beats is

$$i^2_{m2} = 4 P^2 P_s P_{bi} \tag{6}$$

The total mean square current due to noise is,

$$N = 2eI_d \Delta f_m + 2ep\ P_b \Delta f_m + 2ep(P_s + P_r)\Delta f_m + 2ep\ P_{NOB} \Delta f_m$$

$$+ K^2{}_1 \left(\frac{4P^2 P_r P_b \Delta f_i}{\Delta f_o} + \frac{4P^2 P_b P_s \Delta f_i}{\Delta f_o} \right) \tag{7}$$

where K_1 is the gain constant referred to in Section 11.

$$N = 2e\Delta f_m \left(I_d + P\,P_b + P\,(P_s + P_r) + P\,P_{NOB} \right)$$

$$+ \frac{4\,P^2 \Delta f_i\, K^2{}_1\, P_b}{\Delta f_o} (P_s + P_r)$$

Assume $P_r \gg P_b + P_s + P_{NOB}$

$I_d \ll P\,P_r$

$P_s \ll P_r$

$$N = 2\,e\Delta f_m\,P\,P_r + \frac{4P^2 \Delta f_i\, K^2{}_1\, P_b P_r}{\Delta f_o}$$

$$N = 2\,e\,p\Delta f_m P_r \left[1 + \frac{2\,P\Delta f_i\, K^2{}_1\, P_b}{e\,\Delta f_o \Delta f_m} \right] \tag{8}$$

For equal attenuation of signal and noise

$$\frac{2\,P\Delta f_i\, K^2{}_1\, P_b}{e\Delta f_o \Delta f_m} \gg 1 \tag{9}$$

$$p = \frac{e}{hv} \tag{10}$$

Let $hv = 10^{-19}$

$\in = .1$

$\Delta f_i = 10^9$

$\Delta f_m = 10^6$

$K^2{}_1 = .5$

$$\frac{2\epsilon \Delta f_i \, K^2{}_1 \, P_b}{hv \Delta f_o \Delta f_m} \gg 1$$

$$P_b \gg \frac{hv \Delta f_o \Delta f_m}{2 \, Q \Delta f_i \, K^2{}_1}$$

$$P_b \gg \frac{10^{-19}(10^{11})(10^6)}{2(.5)(10^9)(.5)} = \frac{10^{-2}}{5 \times 10^8}$$

$$= 2 \times 10^{-11} \text{ watts}$$

$$S/N = \frac{P_s \Delta f_o}{2 \Delta f_i \ P_b} \tag{11}$$

If

$$\frac{2 \ P \ \Delta f_i \ K^2{}_1 \ P_b}{e \Delta f_o \Delta f_m} \ll 1$$

$$S/N = \frac{K \ P \ P_s}{e \ \Delta f_m} \tag{12}$$

In equation (11), noise due to mixing between the I.O. and background predominates over shot noise. Equation (12) expresses the reverse situation.

The S/N for non-coherent detection is

$$(S/N)_n = \frac{i^2{}_s}{i^2{}_d + i^2{}_b + i^2{}_c}$$

where

$$i^2_s = \frac{P^2 P^2_s}{2}$$

$$i^2_d = 2\ e\ I_d \Delta f_m$$

$$i^2_b = 2\ e\ P\ P_b \Delta f_m$$

$$i^2_c = 2\ e\ P\ P_s \Delta f_m$$

$$(S/N) = \frac{P^2 P^2_s}{4e \Delta f_m\ (I_d + P\ P_b + P\ P_s)}$$

$$I_d \ll P\ (P_b + P_s)$$

$$S/N = \frac{P\ P^2_s}{4e \Delta f_m\ (P_b + P_s)} \tag{13}$$

Comparing (11) and (13)

$$\frac{(S/N)_{c1}}{(S/N)_n} = \frac{2\ hv}{\epsilon} \left(\frac{\Delta f_o \Delta f_m}{\Delta f_i} \right) \left(\frac{P_b + P_s}{P_s P_b} \right) \tag{14}$$

Comparing (12) and (13)

$$\frac{(S/N)_{c2}}{(S/N)_n} = \frac{4K^2_1 (P_b + P_s)}{P_s} \tag{15}$$

The relationships between P_b and P_s may be defined:

1. $P_b > P_s$

2. $P_b \approx P_s$

3. $P_b < P_s$

For (1),

$$\frac{(S/N)_{c1}}{(S/N)_n} = \frac{2\,h\nu}{\epsilon}\left(\frac{\Delta f_o \,\Delta f_m}{\Delta f_i}\right)\left(\frac{1}{P_s}\right)$$

$$\frac{(S/N)_{c2}}{(S/N)_n} = \frac{4K^2{}_1\; P_b}{P_s}$$

For (2),

$$\frac{(S/N)_{c1}}{(S/N)_n} = \frac{4\,h\nu}{\epsilon}\left(\frac{\Delta f_o \,\Delta f_m}{\Delta f_i}\right)\left(\frac{1}{P_{bs}}\right)$$

$$\frac{(S/N)_{c2}}{(S/N)_n} = 8\,K^2{}_1$$

For (3),

$$\frac{(S/N)_{c1}}{(S/N)_n} = \frac{2\,h\nu}{\epsilon}\left(\frac{\Delta f_o \,\Delta f_m}{\Delta f_i}\right)\left(\frac{1}{P_b}\right)$$

$$\frac{(S/N)_{c2}}{(S/N)_n} = 4\,K^2{}_1$$

SECTION 13

CONCLUSIONS
SECTIONS 8 THROUGH 12

Although a final judgement must be deferred until completion of the systems integration phase of the study, certain conclusions can be drawn from current results.

The study has supported our belief that optical communication has the potential to replace radio and to perform unique functions in many space situations. To realize this potential, there must be improvements in all components of the system. The receiving sub-system is one of the most rewarding areas for future effort.

As in radio, it is economically justifiable to develop ground receiving equipment to a high degree of sophistication. Such effort will not only improve the system performance to a significant degree, but will permit important savings in the associated optical equipment in the weight of on-board equipment.

Heterodyne (and homodyne) receivers require that the mixing waves be nearly congruent in the interacting surface or volume. This requirement is so severe that essentially diffraction limited conditions must be attained in the transmission medium and optics. The practical result is that such receivers are restricted to small area collecting optics and in atmosphere, and to high quality optics in any case. Certain methods have been herein proposed to overcome this limitation. If a successful method is adopted the heterodyne principal will be decidedly superior for most applications. Many problems will remain however, principally oscillator stability and noise, and the effects of doppler shift due to vibration and vehicle motion. The quantum counter receiver has only one real disadvantage, the inability to reject incident out of band noise. This disadvantage is important because of the unavailability of optical filters having a bandwidth comparable to the signal bandwidth. However, if the quantum counter receiver is operated in a situation appropriate to the heterodyne wave congruence requirement, i.e., diffraction limited field of view, the quantum counter will give equal or better performance in many practical cases.

It has also been found that for moderate background noise, wideband modulation will partially overcome the noise rejection deficiency of the quantum counter.

SECTION 14

RECOMMENDATIONS
SECTIONS 8 THROUGH 12

On the basis of the work done to date, it is evident that there is no clear cut choice between the alternative methods of detection. A clear preference for a particular approach will develop as the result of further investigation. Enough work must be done in both alternatives to uncover difficulties and advantages which are now unknown. This work must be in part experimental. In addition to the systems analysis which is the final phase of the current contract, we recommend that analytical and experimental programs be initiated on the following subjects:

I. ANALYSIS AND EXPERIMENTAL VERIFICATION OF HETERODYNE DETECTION

The analysis of heterodyne detection which is presented here should be extended and refined to include the effects of oscillator noise, multiple modes, and doppler shift due to vehicle motion and vibration. The analytical results should be verified by experiments designed to provide quantitative information now lacking.

II. ANALYSIS AND DEMONSTRATION OF WAVE CONGRUENCE PHENOMENA AND REMEDIAL MEASURES

The problem of wave congruence in heterodyne detection should be experimentally verified, and proper solutions should receive further analysis. If warranted, the analysis should lead to breadboard demonstrations of principle.

III. IMPROVEMENT OF QUANTUM EFFICIENCY

Means of improving photo-emission quantum efficiency in the long visible and near infra-red wavelengths should be reviewed, particularly the use of multiple pass thin film techniques. Promising methods should be experimentally checked.

IV. ATMOSPHERIC PROPAGATION EXPERIMENTS

Further experimental effort is recommended in atmospheric propagation of laser signals. The tests should include actual heterodyne reception over atmosphere paths, and measurements of intensity modulation and wave distortion in vertical or near vertical atmospheric paths.

APPENDIX I

INFORMATION RATE IN PULSE SYSTEM

I. ERROR RATE COMPUTATIONS (LOW NOISE LEVEL)

Curves of error rate versus signal and noise have been plotted by the following procedure:

1. Select value of noise (N)
2. Evaluate

$$P_{e1} = 1/2 \sum_{r=x}^{\infty} \frac{e^{-N} N^{r}}{r!}$$

This is done by selecting a threshold x for a desired noise error rate P_{e1}.

3. Evaluate

$$P_{e2} = 1/2 \sum_{0}^{x} \frac{e^{-(N+S)} (N+S)^{r}}{r!}$$

based on a threshold, fixed noise, and signal (S) for a desired error rate.

II. ERROR RATE IN LARGE SIGNAL-TO-NOISE SYSTEMS

In this analysis, the Poisson distribution will be approximated by the Gaussian distribution. For this approximation to be valid, the detected noise must be large ($N \doteq 100$). The probability of error for an equal number of received 1's and 0's is

$$P = 1/2 \left[\int_{Y}^{\infty} \frac{e^{\frac{-(x-N)^2}{2N}}}{\sqrt{2\pi N}} dx + \int_{0}^{\infty} \frac{e^{\frac{-(x-N-S)^2}{2(N+S)}}}{\sqrt{2\pi (N+S)}} dx \right]$$

$$P = 1/2 \left[1 - \int_0^Y \frac{e^{\frac{-(X-N)^2}{2N}} dX}{\sqrt{2\pi N}} + \int_0^Y \frac{e^{\frac{-(X-N-S)^2}{2(N+S)}}}{\sqrt{2\pi N}} \right]$$

$$P = 1/2 \left[1 - 1/2 - \int_N^Y \frac{e^{\frac{-(X-N)^2}{2N}} dX}{\sqrt{2\pi N}} + 1/2 - \int_Y^{N+S} \frac{e^{\frac{-(X-N-S)^2}{2(N+S)}} dX}{\sqrt{2\pi (N+S)}} \right]$$

Converting to the normal distribution

$$\emptyset (t) = \frac{1}{\sqrt{2\pi}} e^{-t^2/2}$$

and letting $t_1 = \frac{X-N}{\sqrt{N}}$, $t_2 = \frac{X-N-S}{\sqrt{(N+S)}}$

$$P = 1/2 \left[1 - \int_0^{\frac{Y-N}{N}} \frac{e^{-t_1^2/2} dt_1}{\sqrt{2\pi}} + \int_0^{\frac{Y-N-S}{N+S}} \frac{e^{-t_2^2/2} dt_2}{\sqrt{2\pi}} \right]$$

where Y = threshold. The optimum value is given by,

$$Y = \sqrt{\frac{N (N+S) (S+\ln \frac{N+S}{N})}{S}}$$

III. INFORMATION RATE IN PULSE SYSTEMS

Assume a PCM receiver in which a number of received photons are required in a time slot to define a pulse. The peak received power is,

$$P_s = \frac{S_\rho}{K_1 \mathcal{T}} \left(\frac{\text{photons}}{\text{sec}}\right)$$

Where

S_p = number of detector output photoelectrons
τ = pulse width
K_1 = quantum efficiency

The average power is,

$$\bar{P} = P_S \left(\frac{n\tau}{T}\right)$$

where

n = number of pulses of length τ contained in a time T.

Combining equations,

$$\bar{P} = \frac{S_p}{K_1\tau}\left(\frac{n\tau}{T}\right) + \frac{S_p n}{K_1 T}$$

The probability of error may be expressed as,

$$P_e = \frac{C^1 - C}{C^1}$$

where

C^1 = number of transmitted bits/sec = 2/pulse for PCM
C = number of error free bits/sec or information capacity

then

$$\bar{P} = \frac{S_p C}{2\ K_1 (1 - P_e)}$$

It is possible to take account of error rates for a restricted set of conditions. First, the curves of Sections I and II are used to write an empirical equation for error probability as a function of signal and noise. This equation is

$$\log_{10} P_e = K_2 S_p$$

where K_2 is a negative number which decreases slowly with increasing noise. Some typical values are listed below:

N	K_2
30	-.07
40	-.06
90	-.04
200	-.033
500	-.023

The average power becomes,

$$\overline{P} = \frac{C \log_{10} P_e}{2K_1 K_2 \ (1-P_e)}$$

$$C = \frac{2K_1 K_2 \ \overline{P} \ (1-P_e)}{\log_{10} P_e}$$

Substituting $K_2 \ S_p = \log_{10} P_e$

$$C = \frac{2K_1 \overline{P} \ (1-P_e)}{S_p}$$

An increase in average power can affect the information capacity in two ways. First, more pulses may be sent without changing the error rate. Second, the peak power per pulse may be increased and the error rate will be reduced. Since $P_e < 1$, $|\log P_e|$ will increase with decreasing error rate.

The energy per pulse may be related to the previous equations. The result is,

$$E_p = \frac{\log_{10} P_e}{K_1 K_2} \quad \text{(photons)}$$

It is also possible to define a detector efficiency A, which can be expressed as

$$A = C/\overline{P} = \left(\frac{2K_1 K_2 \ (1-P_e)}{\log_{10} P_e} \right)$$

$$A = \frac{2 \ K_1 (1-P_e)}{S} \ \frac{\text{bits}}{\text{photon}}$$

With a few modifications, the analysis may be extended to PPM. The expression for average power is

$$\overline{P} = \frac{P_{n2}\,\tau}{T}$$

where n_2 is the number of pulses of length τ received in T sec. The information contained in the pulse is

$$I = \log_2 N = \log_2 \frac{T}{\tau}$$

$$\frac{T}{\tau} = \frac{P_{n2}}{\overline{P}}$$

$$I = \log_2 \frac{P_{n2}}{\overline{P}}$$

The information capacity is

$$C_2 = \frac{I}{T} = \frac{1}{T}\left[\log_2 \frac{P_{n2}}{\overline{P}}\right]\left[(1-P_e)\right]$$

$$P = \frac{S}{K\tau} + \frac{\log_{10} P_e}{K_1 K_2 \tau}$$

$$C_2 = \frac{1}{T}\left\{\log_2\left[\frac{n_2}{\overline{P}} \quad \frac{\log_{10} P_e}{K_1 K_2}\right]\right\} (1-P_e)$$

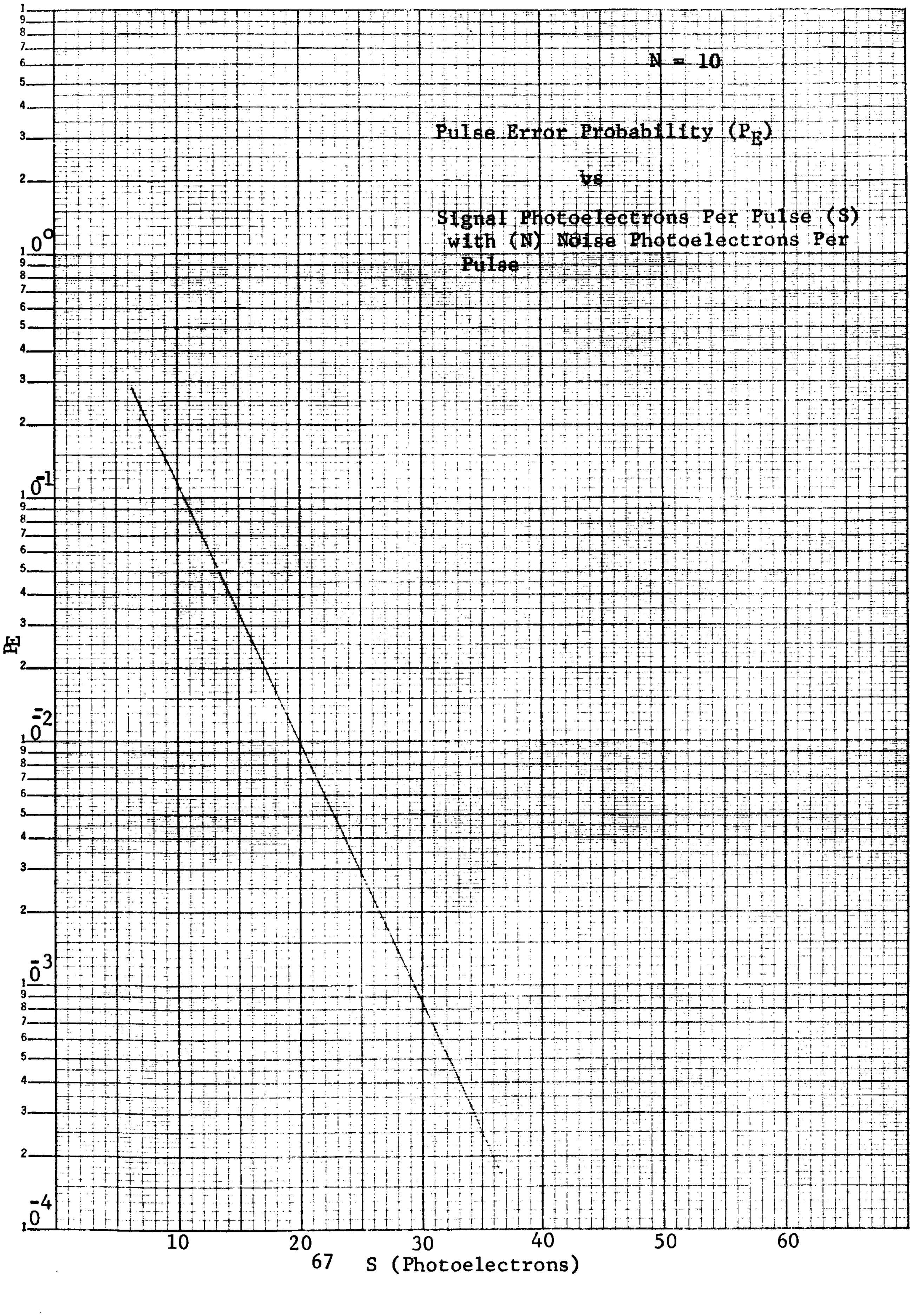
Letting T = 1 sec

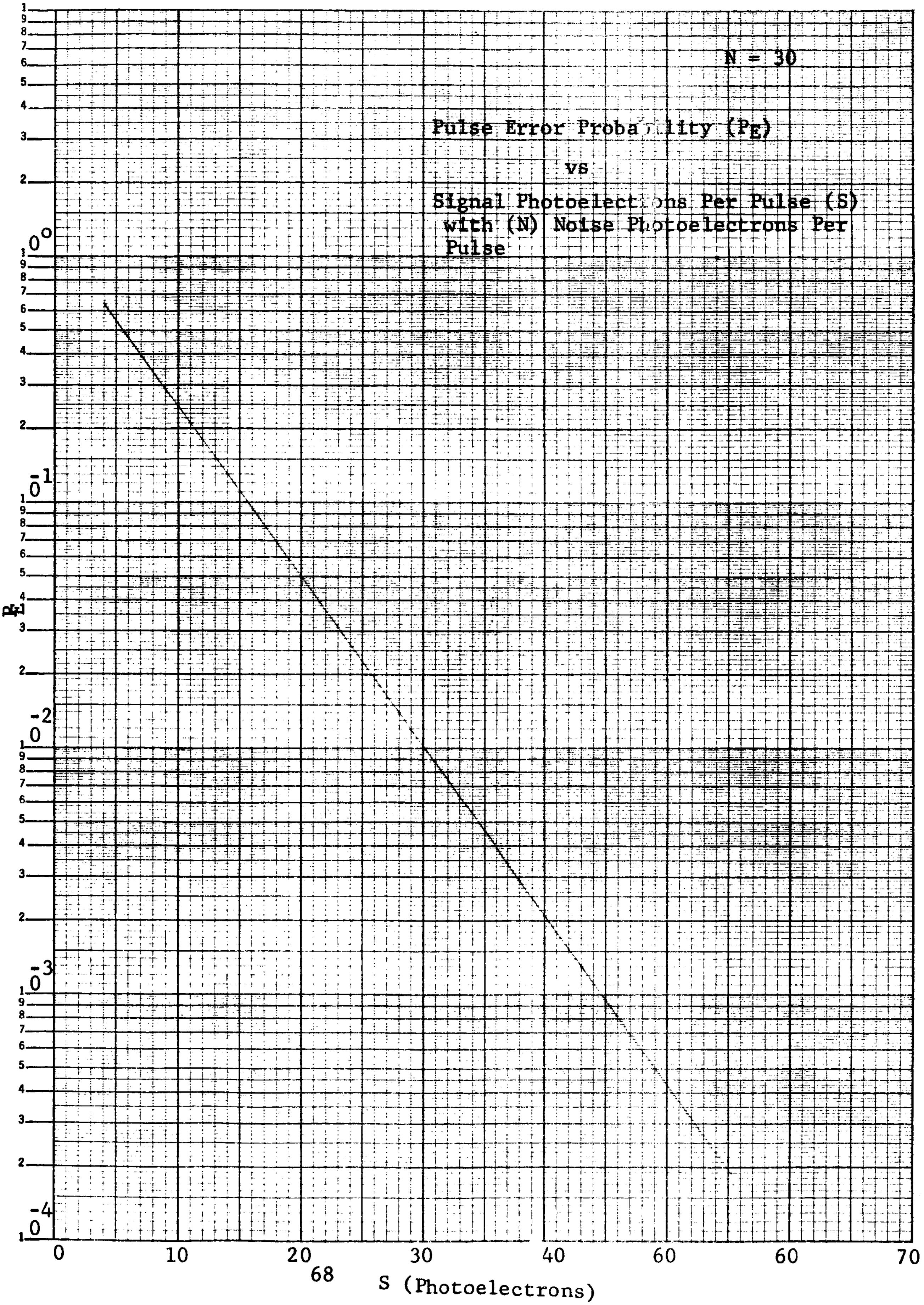
$$C_2 = \left\{\log_2\left[\frac{n_2}{\overline{P}}\,\frac{\log_{10} P_e}{K_1 K_2 \tau}\right]\right\}\left\{(1-P_e)\right\}$$

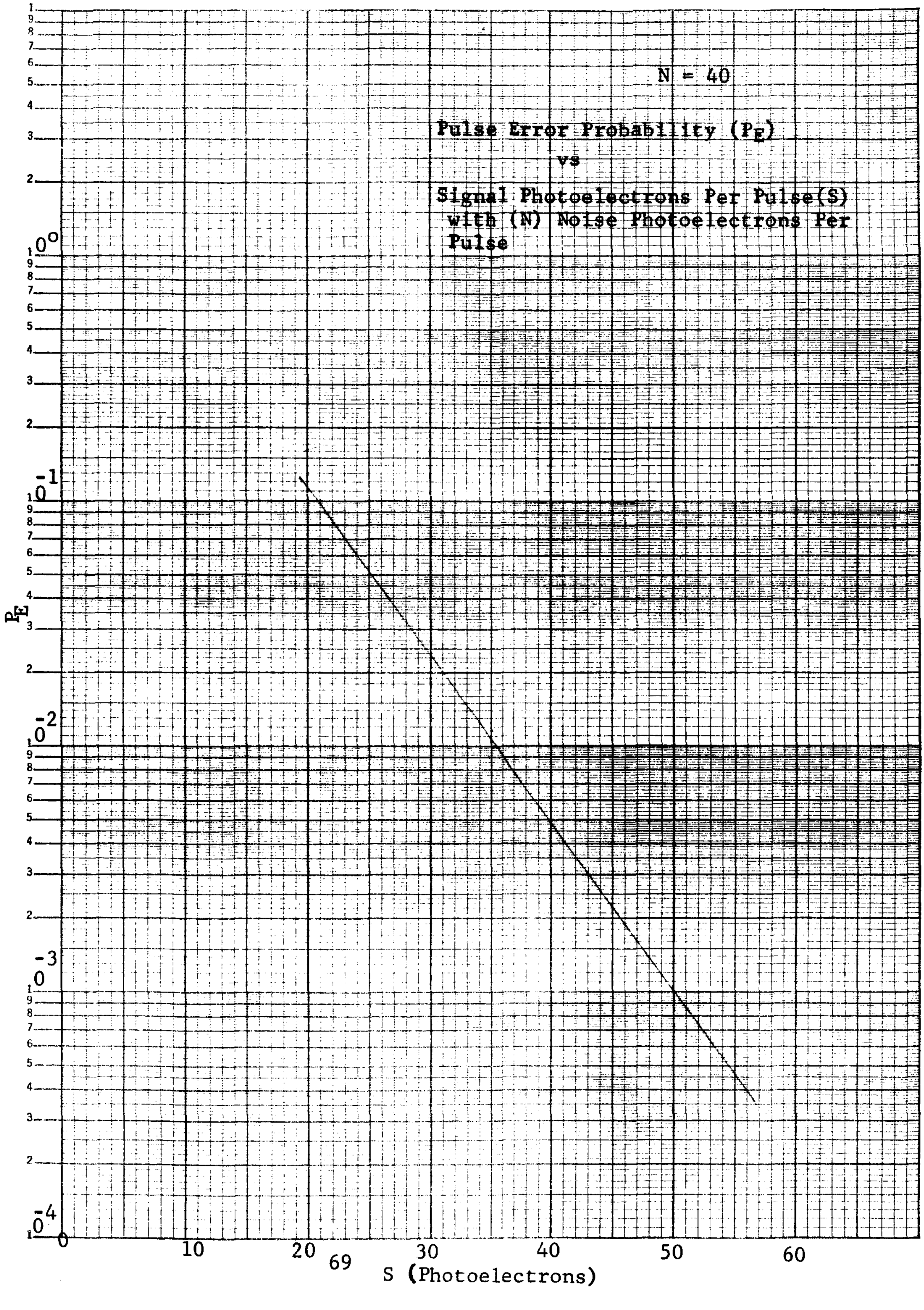
$$C_2 = (1-P_e) \log_2\left[\frac{n_2\, S}{P\, K_1 \tau}\right]$$

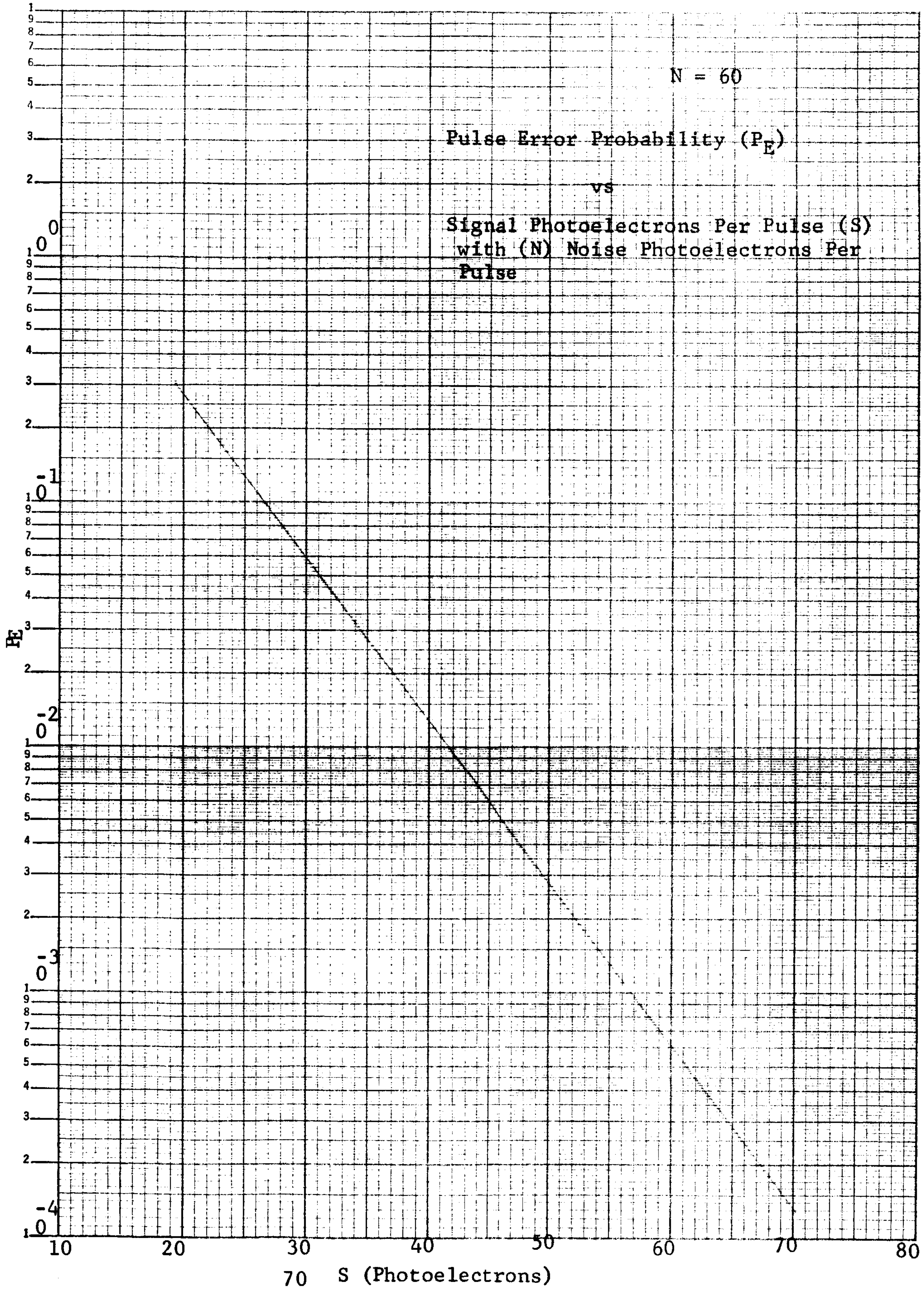
The detector efficiency is

$$A = \frac{K_1(1-P_e)}{Sn_2} \qquad \log_2 \frac{1}{\tau}$$


N = 10
Pulse Error Probability (P_E)
vs
Signal Photoelectrons Per Pulse (S) with (N) Noise Photoelectrons Per Pulse
P_E
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10
20
30
40
50
60
S (Photoelectrons)

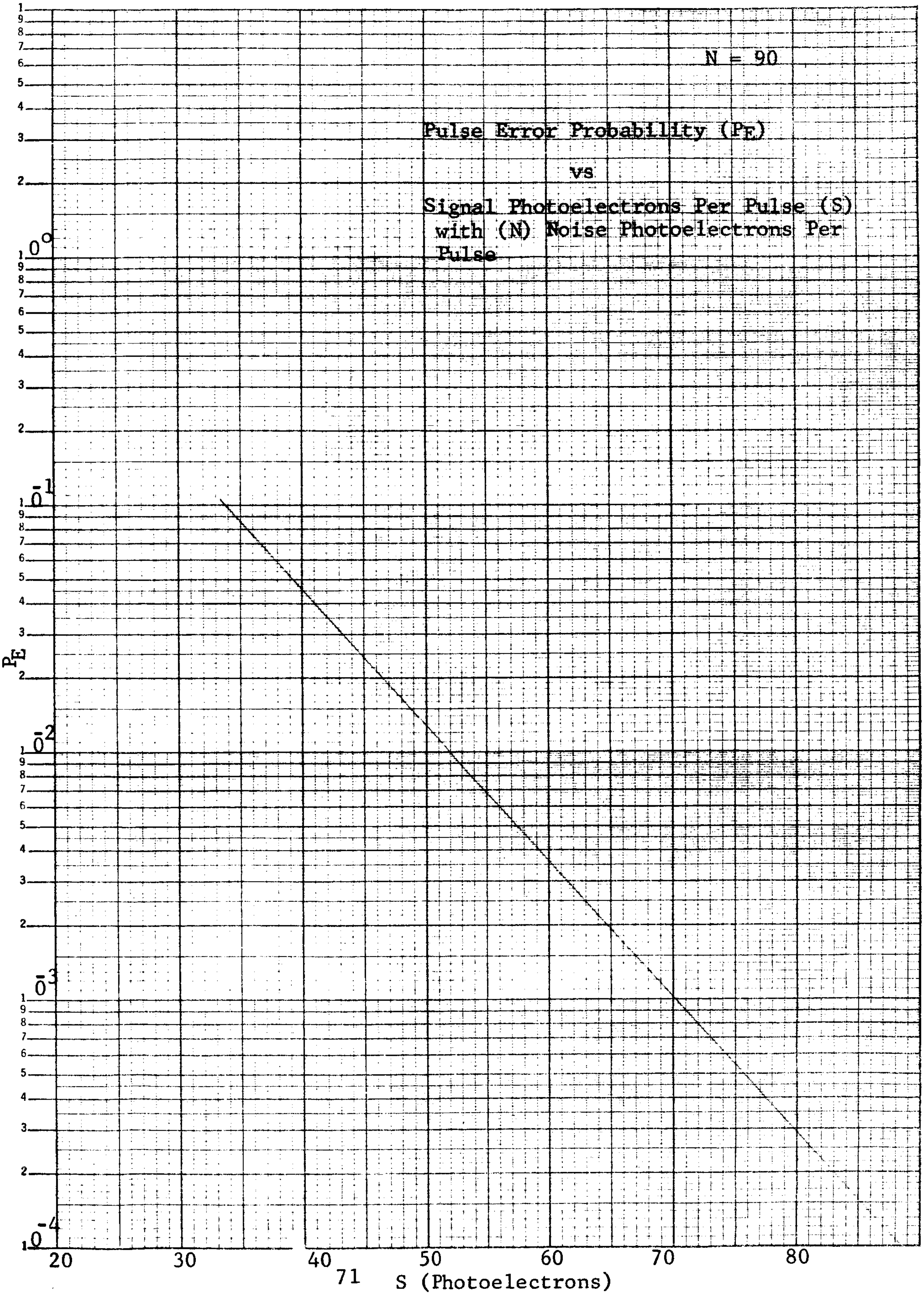

N = 30
Pulse Error Proba??lity (PE)
vs
Signal Photoelect??ons Per Pulse (S) with (N) Noise Photoelectrons Per Pulse
P_E
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
0
10
20
30
40
60
60
70
S (Photoelectrons)

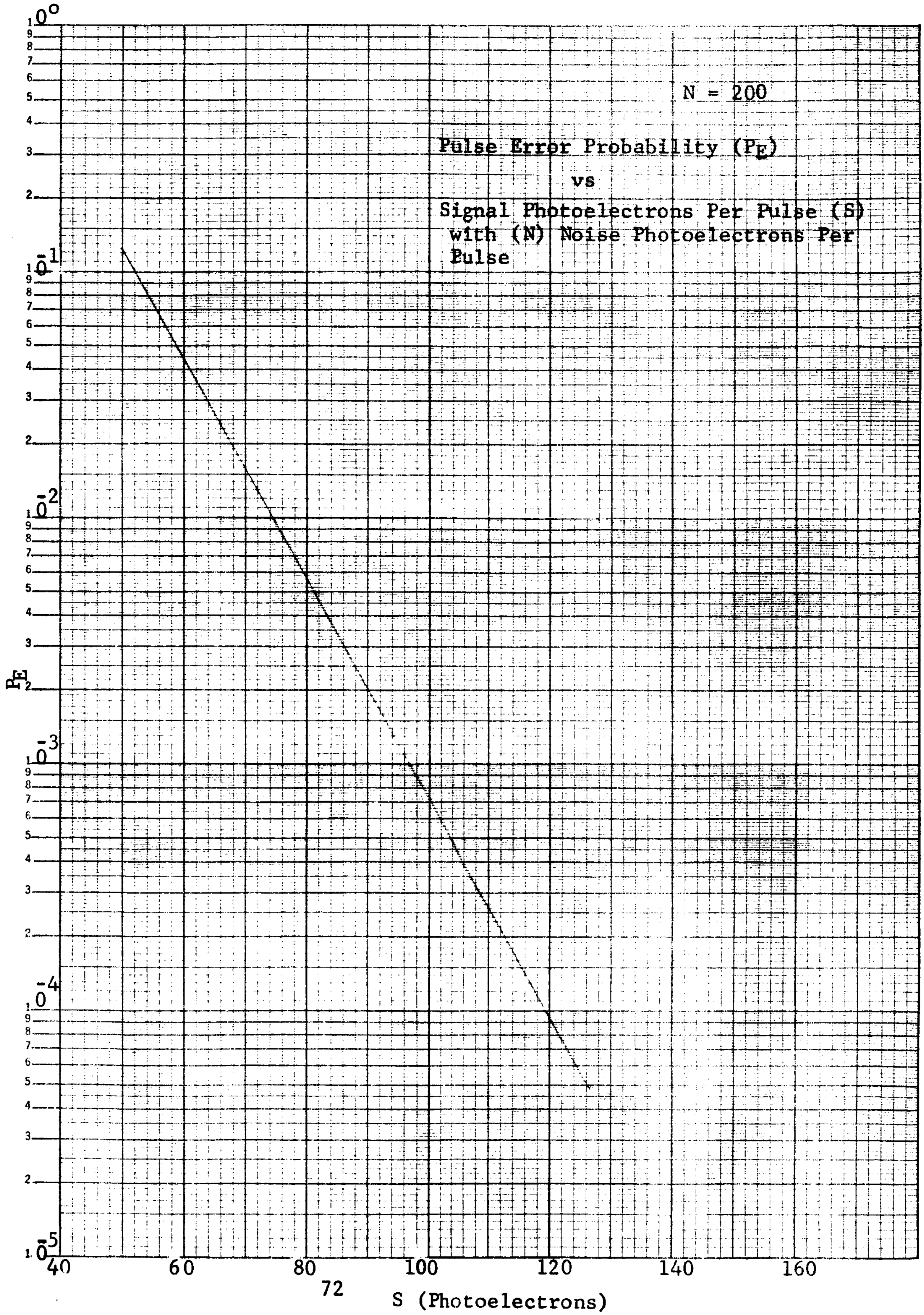

N = 40
Pulse Error Probability (P_E)
vs
Signal Photoelectrons Per Pulse(S)
with (N) Noise Photoelectrons Per Pulse
P_E
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
0
10
20
30
40
50
60
S (Photoelectrons)

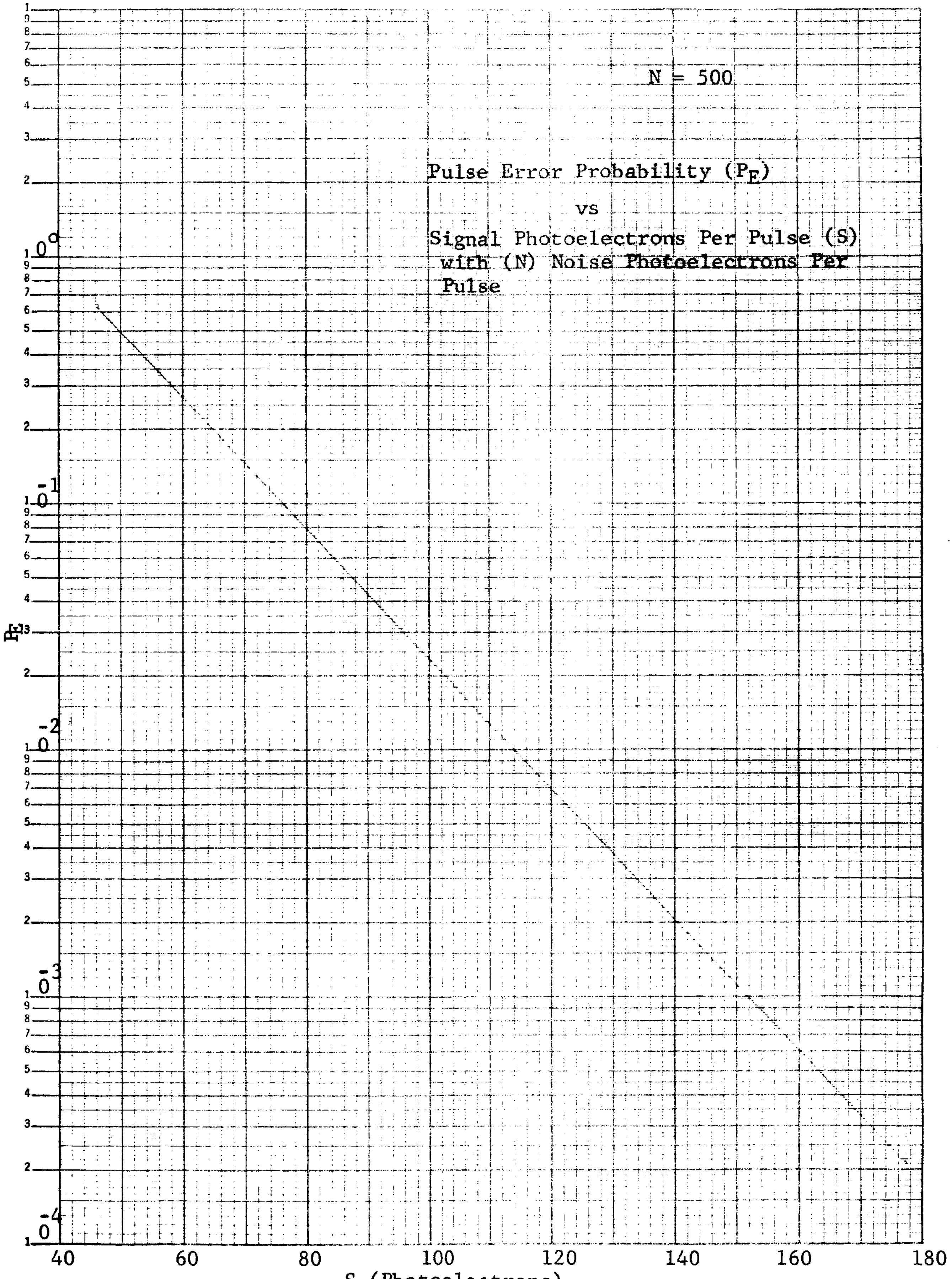


N = 60
Pulse Error Probability (P_E)
vs
Signal Photoelectrons Per Pulse (S)
with (N) Noise Photoelectrons Per
Pulse
P_E
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10
20
30
40
50
60
70
80
S (Photoelectrons)

70


N = 90
Pulse Error Probability (P_E)
vs
Signal Photoelectrons Per Pulse (S) with (N) Noise Photoelectrons Per Pulse
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
P_E
20
30
40
50
60
70
80
S (Photoelectrons)


N = 200
Pulse Error Probability (P_E)
vs
Signal Photoelectrons Per Pulse (S)
with (N) Noise Photoelectrons Per
Pulse
P_E
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
40
60
80
100
120
140
160
S (Photoelectrons)


N = 500
Pulse Error Probability (P_E)
vs
Signal Photoelectrons Per Pulse (S) with (N) Noise Photoelectrons Per Pulse
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
P_E
40
60
80
100
120
140
160
180
S (Photoelectrons)

APPENDIX II

CODING

The following notation will be used:

n = total number of bits in a coded work
K_n = number of message bits in a coded word
p = bit error probability for received coded word
p_r = required bit error probability for decoded word
t = number of errors corrected by the code

$$\binom{n}{j} = \frac{n!}{j!\,(n-j)!}$$

Given a code (n, K_n, t), the error may be expressed as,

$$1 - \sum_{j=0}^{t} \binom{n}{j} q^{n-j} p^{j} = K_n p_r \qquad q = 1-p$$

Taking only the first term of the summation,

$$\binom{n}{t+1} p^{t+1} = K_n p_r$$

As an example of the effect of coding, several numbers will be taken from the curves (Appendix 1). Let

N = 60 photoelectrons
S = 72 photoelectrons
$p = 10^{-4}$

Assume a Wagner code which uses a parity bit to correct one error per word. Also assume that a word consists of 10 message bits. Then,

$$n = K_n + 1$$

$$K_n = 10 \quad n = 11, \quad t = 1, \quad p_r = 10^{-4}$$

Substituting

$$\binom{11}{2} p^2 = 10\ (10^{-4}) \qquad\qquad p = .0043 \ .$$

Therefore, with a bit error probability of .0043, the decoded bit error probability is 10^{-4}. From the curves,

$$p = .0043$$
$$N = 60 \text{ photoelectrons}$$
$$S = 47 \text{ photoelectrons}$$

The average received power is

$$\bar{P} = \frac{DP}{2}$$

where D = duty cycle

and $D = n \left(\frac{\tau_1}{T}\right)$

P = peak power

where

τ_1 = pulse width
T = time required to receive n bits pulses

The factor of two indicates that an equal number of 1's and 0's are received. Then,

$$\bar{P} = \frac{P}{2}\left[n \ \frac{\tau_1}{T}\right]$$

For the uncoded case let,

$$\bar{P}_1 = \frac{P_1}{2}\left[n_1 \frac{\tau_1}{T}\right]$$

With coding,

$$\bar{P}_2 = \frac{P_2}{2}\left[n_2 \ \frac{\tau_1}{T}\right]$$

$$\frac{\bar{P}_2}{\bar{P}_1} = \frac{n_2 \ P_2}{n_1 \ P_1}$$

From the curves,

$$P_1 = 72$$

$$P_2 = 47$$

$n_1 = 10$ which is the uncoded word length. $n_2 = 11$ because a parity bit must be included,

$$\frac{\overline{P}_2}{\overline{P}_1} = \frac{11}{10} \times \frac{47}{72} = 0.72$$

The required energy per pulse will be reduced by a factor $\frac{47}{72} = .66$

The preceding calculations will be repeated for a lower noise level.

Let $N = 10$
$S = 40$
$p = 10^{-4}$

$$\binom{n}{t+1} p^{t+1} = K_n p_r$$

$$n = 11 \qquad p_r = 10^{-4}$$

$$t = 1$$

Solving for p,

$$p = .0043$$

Then $N = 10$
$S = 25$
$p = .0043$
$p_r = 10^{-4}$

$$\frac{\overline{P}_2}{\overline{P}_1} = \frac{n_2\ P_2}{n_1\ P_1} = \frac{11}{10} \times \frac{25}{40} = .69$$

The energy reduction is $\frac{25}{40} = .63$

Calculations will be summarized for other noise levels. The assumptions will be,

$$p = .0043$$
$$p_r = 10^{-4}$$
$$\left.\begin{array}{l} n = 10 \\ K = 10 \end{array}\right\} \text{uncoded}$$
$$\left.\begin{array}{l} n = 11 \\ K_n = 10 \end{array}\right\} \text{coded (1 parity bit)}$$

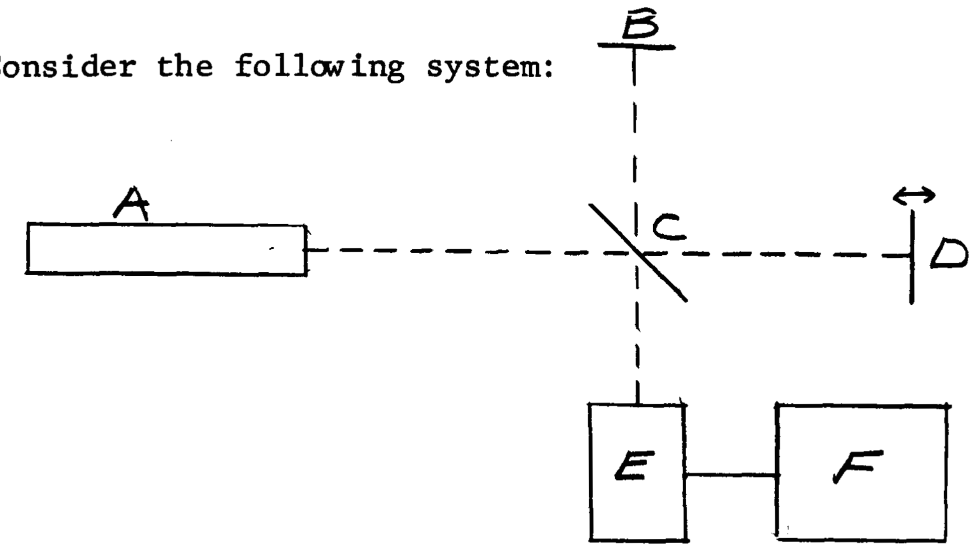
S_1 = required signal without coding
S_2 = required signal with coding

N	S_1	S_2	$\overline{P}_2/\overline{P}_1$	E_2/E_1
30	60	36	.66	.60
50	70	44	.69	.63
80	85	55	.69	.63
90	88	59	.74	.67
200	120	83	.76	.69
500	185	126	.75	.68
1000	250	176	.77	.70

APPENDIX III

Doppler FM

Consider the following system:



Monochromatic collimated light from A strikes a half silvered mirror C. A portion of the light strikes the detector E. The remainder strikes mirror D which has a velocity component parallel to the collimated light. The light is directed to the detector by means of mirror B.

The laser output has been observed to contain a series of modes within the output bandwidth. The frequency spacing between modes is inversely proportional to the laser cavity length. For a 1 meter cavity, this frequency is about 150 mc.

The doppler shift due to motion of mirror D is,

$$\Delta f = \frac{2v}{\lambda}$$

where v = relative velocity
λ = laser wavelength

Assume the mirror has some periodic motion described by

$$a = A_o \cos w_m t$$

where A_o = amplitude

ORIGINAL PAGE IS
OF POOR QUALITY

Then,

$$v = A_o \, w \sin w_m t$$

and

$$|v_{max}| = A_o \, w_m$$

Substituting

$$\Delta f = \frac{2}{\lambda} (A_o \, w_m)$$

In an FM system, the deviation ratio is,

$$B = \frac{\Delta f}{f_m}$$

where f = frequency deviation
f_m = modulating frequency

$$B = \frac{2}{\lambda} \frac{A_o \, w_m}{f_m} = \frac{4 \pi A_o}{\lambda}$$

The narrowband criterion may be applied to this expression. For $B \leq \pi/2$ the receiver input consists of a carrier plus an upper and lower sideband. For $B > \pi/2$, additional sidebands are present. The required received bandwidths are:

$B >> 1$	$B = 2 \Delta f$
$B \approx 1$	$B = 2 \; f_m$

The detector input consists of an unmodulated local oscillator and a frequency modulated signal. Each of these components consists of a series of modes. If wavefront congruency requirements are satisfied, mixing will occur at the photosurface. The expected detector output would be a set of fundamental and harmonic RF components containing the modulation. A conventional FM receiver with adequate bandwidth would be sufficient for detection.

APPENDIX IV

GENERAL COMMENTS ON MODULATION AND DEMODULATION

A. OPTICAL MODULATORS

This section includes a brief description of the various methods of producing modulation on laser beams and of how these modulation methods can be applied to the present problem.

Basically, there are two methods of modulating a light beam; namely, internal and external to the laser material. Internal methods are concerned with modulating the light beam before it emerges from the laser itself so that the modulation of the light beam is produced directly. External methods take the unmodulated light beam produced by the laser and add the modulation after the beam is generated.

B. INTERNAL METHODS OF MODULATION

1. Zeeman Effect

The Zeeman effect can be used to change the energy levels in a laser material. The resulting modulation is limited to fairly small bandwidths for two reasons. In the first place, the Zeeman coefficient of ruby is approximately 2.6 mc per gauss which would produce a corresponding frequency shift for 10 kilogauss of only about 26 mc. Secondly, it is difficult to vary the magnetic field at rapid rates.

2. Stark Effect

This again requires a fairly large coefficient if it is to be used for wideband frequency modulation. For example, in ruby the Stark coefficient is approximately 1.8×10^{-5} cycles centimeters per volt. Thus, with a field of 10^5 volts per centimeter a frequency shift of only about 20 megacycles is obtained.

3. Q Switching

This would produce amplitude modulation. Care must be taken in selecting the kind of modulation to be placed on the beam since it may not be feasible to change the Q at either extremely slow or extremely fast rates. If the laser is held off from lasing too long spontaneous relaxation may occur. On the other hand, if the modulation is too fast, the pumping speed may be insufficient to keep the population levels inverted.

4. Varying the Interferometer frequency

This can be done in a number of ways. Two methods at present that are promising are the use of a birefringent material, such as ADP or KDP, placed at the end of the laser rod and driven by the modulation voltage. This effectively changes the length of the cavity since the plane of polarization is changed by the birefringent material, thereby producing frequency modulation. Care must be taken in using most of these birefringent materials in order that excess heat is not absorbed by the crystal which would cause undesirable strains in the crystal. In addition, for most frequency modulation schemes, the temperature of the crystal itself must be accurately controlled. For example, ruby has a thermal coefficient of expansion of about 8×10^{-6} which would give a frequency shift of 3.7×10^{9} cycles per degree C.

The second method of changing the effective length of the interferometer is by placing a piezo electric material at one end of the crystal. Ultrasonic waves at the modulation frequency form an optical grating since the optical index of refraction, which depends upon the density of the material, is varied by the acoustical wave passing through it. The major problem in using this type of ultrasonic modulation is the transition from the r-f waves to the acoustical wave. Present methods of accomplishing this are fairly inefficient.

5. Change of Bias

In solid state laser diodes, changing of the bias placed across the diode can modulate the output light. This method has not only been shown recently to be feasible, but has the advantage of requiring a very low modulation power as compared to the above methods.

C. EXTERNAL METHODS OF MODULATION

1. Pockels Effect

In this method, the light is passed through an electro-optical birefringent material such that the plane of polarization can be changed by varying the voltage applied to the crystal. This produces polarization modulation, but by the use of polarizers amplitude modulation can be obtained. At present, this has the disadvantage of requiring a very large voltage, usually in the order of kilovolts, to change the polarization by 90°. It has the advantage of operating over fairly wide modulation bandwidths. Using traveling wave structures which can give a cumulative interaction with the light, the power required can be minimized to the order of watts.

2. Piezoelectric Effect

In this case, the light could be either reflected or transmitted through a piezoelectric plate such that either amplitude or phase modulation can be obtained. Disadvantages are that the bandwidth is small and the modulation power required is fairly large.

3. Other Methods

This would include such things as parametric crystals, ferromagnetic materials, use of other laser beams, etc. all of which have been reported on but at the moment are not far enough along in their development to be seriously considered for systems applications.

D. APPLICATION TO PRESENT MISSION

At the moment, for the present mission amplitude modulation would be used with a fairly narrow bandwidth, roughly of the order of tens of megacycles, so that present modulators can easily produce the desired modulation and bandwidth. The only limitation involved would be the limitation on power required which presently is at the watt level for the more promising Pockels effect modulators. However, with some of the advances being made with the Gallium-Arsenide lasers, modulation powers may be reduced considerably. The conclusion that can be reached for the present mission is that modulation does not seem to be a problem since presently available modulators can produce the type of modulation desired at the necessary bandwidths.

E. OPTICAL DEMODULATORS

GENERAL

In many respects, demodulation presents the more difficult of the two problems, i.e., of modulation and demodulation. However, it is difficult to compare the two since the problems are quite dissimilar. With modulation the principal problems are associated with achieving adequate bandwidth and with minimizing the power required to modulate a given light beam. (As noted previously, modulation bandwidth does not represent a real problem in the present application). In demodulation, the principal problem is one of noise. The information capacity of a given communication link is finally limited by the signal-to-noise ratio achieved after final detection. The receiver must be capable of processing the signal with maximum efficiency adding a minimum of noise to that already inherent in the incoming signal.

The two principal means whereby photodetection may be accomplished are by photoemission and by photoconduction. A photoemissive material is one which emits electrons in proportion to the intensity of the incident light. The photo conductor, on the other hand, exhibits a change in resistivity which is proportional to changes in light intensity; in the photoconductor, the action of the light is to excite electrons into the conduction band where they can move freely and carry a current.

Both the photoemitter and the photoconductor respond to radiation intensity (i.e., to the square of the electric field intensity) and are therefore nearly perfect square law devices. This means that either one may be used as a non-linear element to detect interfering beats between two separate waves (as in heterodyning) in addition to being used for direct detection of either coherent or non-coherent light.

Actual light demodulators take on a wide variety of physical forms and employ numerous types of light sensitive materials depending upon the specific applications for which they are to be used. Photoconductive devices, due to their typical junction capacitances, are often resonated in a microwave structure as a means of increasing their impedance level at microwave frequencies. Photoemissive elements may be used directly as a diode or in combination with an amplifier mechanism such as in a photomultiplier tube or in a TWT phototube.

BASIS FOR COMPARING PHOTODETECTORS

The ultimate goal in a light detector is to be able to detect the minimum possible signal power in an incoming light beam. Both the noise generated in the photodetector element and the noise developed in the amplifier following photodetection are important.

Optical receiver sensitivities can be compared on a signal-to-noise basis in a manner analogous to the noise figure definition for ordinary receivers. Instead of an inherent noise power of kTB as at microwave frequencies, the dominant noise power term at light frequencies is hvB. This result may be derived from the total noise spectral density x(v) given as

$$x(v) = \frac{hv}{e^{hv/kT} - 1} + hv \qquad (1)$$

by recognizing that $hv/kT >> 1$ for room temperature and for optical frequencies. As a consequence of the hvB noise inherent in any light signal (caused by the statistical fluctuation associated with quantum flow - a quantity which can be termed radiation shot noise) the maximum possible signal-to-noise ratio of a light signal is

$$S/N = P_S/hvB \qquad (2)$$

where P_S is the signal power and B is the bandwidth in cycles per second. In an ideal optical receiver which adds no noise to the incoming signal, the output signal-to-noise ratio is also given by Equation (2). The deterioration in S/N from that which could ideally be achieved in an optical detection system forms the basis for comparing various photodetection schemes.

COMPARISON OF PHOTOEMITTER VS. PHOTOCONDUCTOR PERFORMANCE

It is difficult to make an absolute comparison between photoemitter and photoconductor detection without considering specific applications or mission requirements. However, from a fundamental point of view, a number of general observations can be made.

In the case of heterodyne detection, where a conversion gain can result in the limiting noise being set by the shot noise of the current leaving the converter, the limiting detectability of both the photoemitter and the photoconductor is given by

$$\frac{S}{N} = \frac{\epsilon n_s}{2\Delta f} \quad \frac{\epsilon P_s}{\epsilon \nu B} \qquad (3)$$

where ϵ is the quantum efficiency of the device, n_s is the number of signal photons per second and Δf is the detector bandwidth. (For heterodyne detection, the signal bandwidth, B, is twice Δf, the detector bandwidth). The above S/N is only achieved when the conversion gain and/or the local oscillator power is sufficient to cause the shot noise to be the dominant noise term in the output of the detector. Under these conditions, the photon-type noise overrides any noise due to "dark" current or thermal noise and the optimum S/N is degraded only by the quantum conversion efficiency.

The quantum efficiencies, ϵ, of semiconductor photoconductors is generally much higher than for photoemissive materials and approaches unity in many materials. On the other hand, the background noise due to dark current is much larger in a photoconductor material and the generally low impedance of these devices makes the power conversion efficiency quite low.

For direct envelope detection (i.e., photon counters) the photo emitter may be far superior to the photoconductor. At very low signal levels, the photoconductor will invariably be limited by thermal and dark current noise whereas the photoemitter can still be essentially shot noise limited provided sufficient gain is built into the device preceding the amplifier which follows the phototube. This gain can be either by electron multiplication or by interaction with a high impedance circuit as in a TWT phototube. No such built-in amplification mechanisms are presently available with photoconductors. (however, in the case of heterodyning the conversion gain acts as a built-in amplification and is one of the advantages of this method of detection.

F. HETERODYNING AS A MEANS OF OPTICAL DETECTION

Certainly, one of the most interesting and promixing means of detecting optical signals is by heterodyning. This method has aroused considerable interest in recent years and has been discussed at great length in the literature. Some of the salient features associated with optical heterodyning are summarized in the following paragraphs.

In optical heterodyne detection, an incoming coherent optical signal is mixed with a coherent local oscillator light signal to produce an interference upon the surface of a non-linear detector element. The resultant difference frequency component of photocurrent can be shown to be proportional to the square root of the product of the local oscillator and the incoming signal power, as in normal heterodyne receivers.

This sensitivity of such a device using coherent light sources can be made to approach that of an ideal amplifier as expressed by Eq. (2). The actual sensitivity is degraded by the quantum efficiency of the photodetector, as indicated in Eq. (3). Quantum efficiencies typically vary from a few percent or less for photoemitters to nearly unity for photoconductors. Quantum efficiencies of photoemitters can theoretically approach 50% for thin layers.

I-F filtering permits the response of the receiver to be limited to the I-F bandwidth of the receiver so that only background noise within this band is amplified. This permits much narrower filtering than is possible with optical filters. From Eq. (3), it is seen that S/N may be increased in direct proportion to the narrowness of the receiver bandwidth.

In addition to filtering, since heterodyne action depends upon the interference between two coherent light signals, this scheme will tend to discriminate against random noise signals originating from non-coherent light sources. By making the local oscillator signals strong enough the signal-to-noise ratio becomes independent of incoherent noise sources, including thermal noise sources, and is limited only by the shot noise of the resulting photocurrent. The interesting and important fact that results from the analysis of optical heterodyning is that the statistical fluctuation associated with the generation of photoelectrons is the exact equivalent of the inherent randomness that is found in a light signal as a result of the quantum nature of light. In other words, the shot noise in the resultant electron beam is equivalent to the radiation shot noise of a light signal, and for unity quantum efficiency the S/N in the photocurrent is the same as the S/N of the incident light. The heterodyne action for a strong local oscillator beam, as a result of the conversion gain associated with the photo mixing process, is to increase the I-F signal power at the same rate as the shot noise from the local oscillator increases. This results in the signal-to-noise ratio being independent of local oscillator strength once the shot noise level exceeds the level of all other noise sources. (This is only true if the local oscillator has no other noise components than radiation shot noise).

In a device such as the TWT phototube the large d-c current level resulting from a large local oscillator signal, besides preserving the incident signal-to-noise, increases the interaction efficiency of the beam with the following microwave circuit element.

Heterodyning also produces a receiver system with a very high degree of directivity. This results from the fact that the local oscillator and signal light must arrive at the photo mixing surface with parallel wave fronts in order to mix properly. If they do not arrive with parallel wave fronts, there will be a phase variation in the beat signal that is produced across the photo mixing surface. These phase differences will tend to cancel each other such that the resultant signal level in the photoelectron current is reduced. A (sin x) /x type of response is generated in which the first zero occurs when the radian measure of the angle between the signal and the local oscillator beam is approximately λ/D, where λ is the optical wavelength and D is the diameter of the receiver aperture over which mixing of the light beams takes place. (In other words, $x = \frac{\pi D}{\lambda} \theta$ where θ is the angle between the two signals.)

Depending upon the aperture diameter, this angle may be very small representing very narrow receiver look angles.

It can also be shown that for very small signal-to-noise ratios ($S/N \ll 1.0$) the heterodyne method of coherent detection still preserves the input signal-to-noise ratio (to within a factor of the quantum efficiency) whereas a straight square law detector, which essentially squares the input signal-to-noise ratio under these conditions will have the output signal-to-noise ratio, degraded by a factor equal to the reciprocal of the input signal-to-noise ratio.

G. DISADVANTAGES OF THE OPTICAL HETERODYNE SYSTEM

The main difficulty with the optical heterodyne system appears to be the problem of maintaining adequate alignment between the incoming signal and the local oscillator signal. Since heterodyning results from an interference phenomenon it is necessary that the two signals be aligned in polarization as well as with respect to the angle between the propagation vectors of the two waves. While this does not appear to be an insurmountable difficulty there does not exist any adequate solution to this problem at the present time for receiver apertures of any size.

It must also be pointed out that any excess noise in the local oscillator signal will add to the noise level of the beat signal in any heterodyne system. Thus, a local oscillator with a relatively pure mode of operation is required. Generally, the oscillation noise will only be a problem for very large local oscillator signal levels. Since local oscillator noise can be shown to increase with the oscillation power an optimum level will ordinarily exist for best signal-to-noise performance.

There is also the problem of tuning the local oscillator to keep the signal I-F frequency with the I-F band of the receiver. In this respect it is interesting to note that at a relative velocity of only approximately twice the speed of sound (1550 mph) a doppler shift in frequency of 1Gc would be observed with red (Ruby) light.

H. CONCLUSIONS

The optical heterodyne receiver appears to be the most promising means of obtaining high sensitivity provided the problem of optical alignment between the signal and local oscillator light beams can be solved adequately. However, if the signal does not remain coherent over the transmission path to the receiver much of the advantage of heterodyning is lost.

It is desirable to employ a device for optical detection which has a built-in amplification mechanism in order that the signal-to-noise ratio is not limited by thermal noise of the following amplifier. Both heterodyning and photoemissive devices are capable of accomplishing this.

There is some advantage in working at the longer wavelengths with respect to the inherent signal-to-noise ratio resulting from photon noise. It is interesting to note that for Ruby light (6943 Å) hv noise is equivalent to kT noise at a temperature of 20,800°K. Thus, in terms of equivalent noise figure, even the "ideal" receiver at light frequencies is extremely noisy. This noisiness increases with the light frequency.

With the possible exception of signal-to-noise ratios that are very much less than unity, the signal-to-noise output of a photoemissive device used as a non-coherent detector approaches that of the superheterodyne case (assuming equal bandwidths). If the dark current is negligible, or if it is reduced by cooling the photoemissive surface to a low temperature, the only real advantages of the superheterodyne system is its ability to discriminate against incoherent background noise and to achieve narrower bandwidth reception than is possible to do optically. Since spatial filtering can be achieved in the case of non-coherent detection by simple optical means, the only real disadvantage is its inability to discriminate against non-coherent background radiation noise originating from the same direction as the signal source, and to filter out noise which is outside of the I-F bandwidth of the superheterodyne receiver and yet within the optical passband of the device. In view of the doppler shifts which can be expected in typical communication missions, the advantages of narrow band reception in the heterodyne case can only be realized when the local oscillator can be made to track the signal frequency with a great deal of precision. However, with tunable lasers it would be possible to construct a system which would either track or phase lock with the signal carrier frequency for either heterodyne or homodyne operation. (A factor of two improvements in signal-to-noise ratio is theoretically achieved with homodyne operation by virtue of the fact that the detector bandwidth need only be half as great as for heterodyne detection).

NOMENCLATURE

C = Information capacity (bits/sec)
B = Bandwidth (cycles/sec)
N = Average noise power (watts)
P_s, S = Average signal power (watts)
E = Wave energy (joules)
h = Planck's constant = 6.63×10^{-34} joule sec
$\underline{r}$, v = Frequency (cycles/sec)
m = Average number of quanta per mode
H = Entropy of information per mode
C_w = Wave information capacity (bits/sec)
R = Entropy rate (bits/sec)
C_h = Information capacity of a heterodyne detector (bits/sec)
K, = Quantum efficiency
ρ_o = Noise spectral density (watts/cycle)
P_r, P_{LO} = Local oscillator power (watts)
P_{NO} = Local oscillator power outside the I.F. bandwidth
B_O = Optical filter bandwidth (cycles)
C_{ho} = Information capacity of a homodyne detector (bits/sec)
C_c = Information capacity of a quantum counter (bits/sec)
E_s = Signal carrier amplitude
E_o = Local source amplitude
τ = Delay between signal and local source zero crossings at a point y on the photosurface
α = Angle between signal and local source waves
Y = Length of photocathode
c = Velocity of light
w_o = Local source frequency (cps)
w_s = Signal carrier frequency (cps)
w_m = Modulating frequency (cps)
K_1 = Gain constant
I_m = Total difference frequency current
λ = Wavelength (meters)
D_o = Entrance aperture (cm)
r_i = Number of sub-collectors
$i^2 d$ = $2\, e\, I_d \Delta f_m$
$i^2 b$ = $2\, e\, p\, p_b \Delta f_m$
$i^2 c$ = $2\, e\, p\, p_{nr} \Delta f_m$
i^2 = $2\, e\, p\, (P_s + P_r) \Delta f_m$
$i^2{}_{nr}$ = $2\, e\, p\, p_{nr} \Delta f_m$

I_d = Dark current

P_b = Background noise power

P_{noB} = L. O. Noise power within the I. F. bandwidth

Δf_m = Modulating frequency bandwidth

e = Electronic charge = 1.6×10^{-19} coulombs

$p = \frac{\epsilon}{h v}$

Δfi = Noise component separation - I.F.

Δfo = Optical frequency bandwidth

P_{e1} = Probability of mistaking noise for signal

P_{e2} = Probability of mistaking signal for noise

x = Receiver threshold

Y = Optimum receiver threshold

P = Peak received power (watts)

S_p = Number of detector output photoelectrons per pulse

τ_1 = Pulse width (sec)

r_1, n_2 = Number of pulses received in a time T

P_c = Probability of error

C^1 = Number of transmitted bits/sec

C = Number of error-free bits/sec

K_2 = Empirical function of noise

E_p = Energy per pulse (photons)

A^p = Pulse detector efficiency

n = Total number of bits in a coded word

K_n = Number of message bits in a coded word

p = Bit error probability for received coded word

P_r = Required bit error probability for decoded word

t = Number of errors corrected by the code

D_o = Diameter of primary collector

o = Field of view of collecting optics

I_o = Incident noise spectral radiance.

BIBLIOGRAPHY

1. Gordon, J. P., "Quantum Effects in Communications Systems" Proc. IRE, Page 1898, September 1962.

2. Stern, T. E., "Some Quantum Effects in Information Channels" IRE Transactions on Information Theory, Page 435, September 1960.

3. Oliver, B. M. "Signal to Noise Ratios in Photoelectric Mixing", Proc. IRE, Page 1960, December 1961.

4. Mayer, H., "Multiple Layer Photocell", Zeit F. Physik, Page 326, Volume 12, 1948.

5. TRG "Technical Note on Heterodyne Detection in Optical Communications", RADC-TDR-62-491, November 3, 1962.

6. Biernson, G., and Lucy, R. F., "Requirements of a Coherent Laser Pulse Doppler Radar", Proc. IEEE, Page 202, January 1963.